MATH 120
Exam 1 Study Guide

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Contents

1.1 Exam 1 Information

NOTE: This guide (and the study guide for the other exams) is not meant to be an exact representation of exam material. I always reserve the right to ask (a small number of) questions that use the course material in a creative way.

Exam 1 content

Exam 1 covers Chapters 1 and 2 in the 2023 version of my MATH 120 lecture notes.

Exam 1 tasks

As always, items marked $\boxed{\text{NC}}$ are those you might have to do without a calculator.

1. $\boxed{\text{NC}}$ Answer questions about course vocabulary

2. $\boxed{\text{NC}}$ Answer questions that use function notation
3. $\boxed{\text{NC}}$ Answer questions that involve substitution into functional expressions **(this task is new as of 2023 and is not on old exams from 2022 or earlier)**
4. $\boxed{\text{NC}}$ Plot vectors in standard position
5. $\boxed{\text{NC}}$ Perform vector addition and scalar multiplication using pictures
6. $\boxed{\text{NC}}$ Draw an angle in standard position
7. $\boxed{\text{NC}}$ Convert angles from radians to degrees, if the radian measure is $\dfrac{A\pi}{B}$ with $B = 1, 2, 3, 4, 6$
8. Solve simple equations like those in Section 1.2 of the lecture notes
9. Perform unit coversions
10. Compute sums, differences, dot products and magnitudes of vectors
11. Compute work using dot products **(this task is new as of 2023 and is not on old exams from 2022 or earlier)**
12. Solve "angle pictures" and story problems that involve angle addition, right and straight angles, complementary and supplementary angles, parallel lines and transversals, angles in triangles, etc.
13. Find a missing side length in a right triangle using the Pythagorean Theorem
14. Classify a triangle as right, acute or obtuse using the Pythagorean Theorem **(this task is new as of 2023 and is not on old exams from 2022 or earlier)**
15. Find angles which are coterminal with a given angle
16. Find an angle between $0°$ and $360°$ which is coterminal with a given angle
17. Questions involving angle reflections and symmetric angles **(this task is new as of 2023 and is not on old exams from 2022 or earlier)**
18. Determine the coordinates of a point on the unit circle with given properties or at a given angle
19. Convert between revolutions, degrees and radians
20. Find circumferences, arc lengths and sector areas
21. Solve problems involving linear and angular velocity (gears and pulleys, etc.)

Facts and formulas to memorize for Exam 1

Dot product: If $\mathbf{v} = \langle a, b\rangle$ and $\mathbf{w} = \langle c, d\rangle$, then $\mathbf{v}\cdot\mathbf{w} = ac + bd$.

Magnitude: If $\mathbf{v} = \langle a, b\rangle$, then $|\mathbf{v}| = \sqrt{a^2 + b^2}$.

Work: $W = \mathbf{F}\cdot\mathbf{d}$.

Pythagorean Theorem: If $\triangle ABC$ is a right triangle, then $a^2 + b^2 = c^2$.

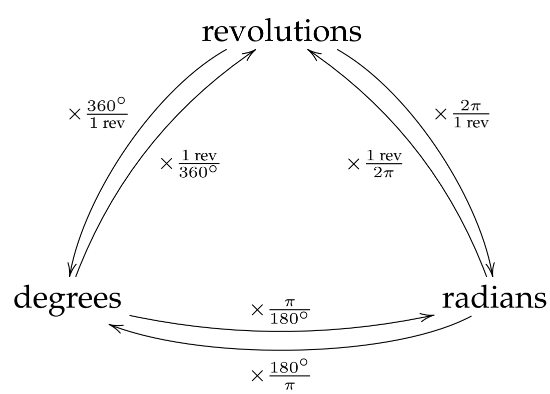
Reflections: If (x, y) is on the terminal side of θ in standard position, then the following angles have the following points on their terminal sides in standard position:

$$\begin{array}{rcll}
\theta & \longleftrightarrow & (x, y) & \\
180^\circ - \theta & \longleftrightarrow & (-x, y) & \text{(reflect } \theta \text{ across the } y\text{-axis)} \\
\theta \pm 180^\circ & \longleftrightarrow & (-x, -y) & \text{(opposite to } \theta\text{)} \\
-\theta \sim 360^\circ - \theta & \longleftrightarrow & (x, -y) & \text{(reflect } \theta \text{ across the } x\text{-axis)} \\
90^\circ - \theta & \longleftrightarrow & (y, x) & \text{(reflect } \theta \text{ across diagonal } y = x\text{).}
\end{array}$$

Unit circle equation: $x^2 + y^2 = 1$

Circumference: $C = 2\pi r$

Conversions between degrees, radians, and revolutions:



Special angle conversions: $\pi = 180^\circ$ $\quad \dfrac{\pi}{2} = 90^\circ$ $\quad \dfrac{\pi}{3} = 60^\circ$ $\quad \dfrac{\pi}{4} = 45^\circ$ $\quad \dfrac{\pi}{6} = 30^\circ$

Arc length formula: $s = r\theta$, so long as θ is in radians

Sector area formula: $A = \dfrac{1}{2}r^2\theta$, so long as θ is in radians

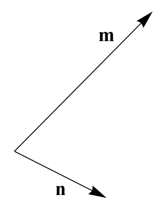
Conversion between linear and angular velocity: $v = r\omega \quad \omega = \dfrac{v}{r}$

1.2 Fall 2023 Exam 1

1. (2.9) $\boxed{\text{NC}}$ Convert each angle from radians to degrees:

   a) $\frac{2\pi}{3}$ b) $-\frac{11\pi}{6}$ c) π d) $\frac{7\pi}{2}$

2. (2.4) $\boxed{\text{NC}}$ Vectors $\mathbf{m}$ and $\mathbf{n}$ are pictured below. On the same picture, sketch the vectors $-3\mathbf{n}$ and $2\mathbf{m}+\mathbf{n}$, carefully labelling your answers so I know which is which.



3. $\boxed{\text{NC}}$ Sketch each angle in standard position. (Make sure to indicate the direction of rotation and the correct number of revolutions.)

   a) (2.9) $\frac{3\pi}{4}$ b) (2.8) $-13°$ c) (2.8) $262°$

4. $\boxed{\text{NC}}$ (2.5) Find an angle between $0°$ and $360°$ which is coterminal with $800°$.

5. (1.3) Throughout this problem, let golf be the function golf $x = x^2 + 2$. Evaluate each expression:

   a) 2 golf 3
   b) golf $^2\,1$
   c) golf $2 - 5$
   d) golf 2 golf 5
   e) golf $2 \cdot 5$

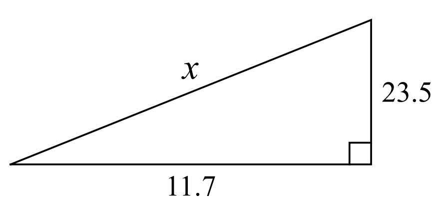
6. a) (1.3) Suppose alpha and bravo are functions so that

   $$3\ \text{alpha}^2 x + \ \text{bravo}^2 x = 30.$$

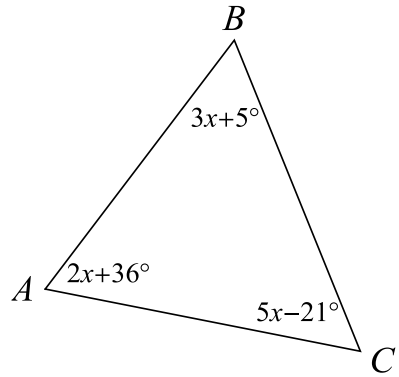
   Substitute alpha $x = 2$ into this equation, and then solve for all possible values of bravo x.

   b) (1.2) Solve for $\cos x$, if $12^2 = 5^2 + 9^2 - 2(5)9\cos x$.

   c) (2.7) Determine the length marked x in this picture:



7. a) (2.6) Find the measure of all three angles of triangle ABC, pictured below:



b) (2.6) Which of these three words best describes the triangle ABC described in part (a)?

ISOSCELES EQUILATERAL SCALENE

c) (2.7) Which of these three words best describes the triangle ABC described in part (a)?

ACUTE RIGHT OBTUSE

8. In this question, suppose $\mathbf{v} = \langle 5, -2 \rangle$ and $\mathbf{w} = \langle -3, 4 \rangle$.

a) (2.4) Compute $3\mathbf{v} + 2\mathbf{w}$.

b) (2.7) Compute the magnitude of $\mathbf{w}$.

c) (2.4) Compute $\mathbf{v} \cdot \mathbf{w}$.

d) (2.3) Sketch $\mathbf{w}$ on a set of (x, y)-axes, so that it starts at the point $(3, 2)$. (Your vector needs to end at the correct spot.)

9. a) (2.10) Compute the area of a circular sector taken from a circle of radius 5.2 ft, if the central angle of the sector measures $40°$.

b) (2.9) Convert $1733°$ to radians.

c) Suppose that a bug is on the edge of a merry-go-round so that it makes 15 revolutions per minute.

i. (2.5) How long will it take the bug to rotate through an angle of $800°$?

ii. (2.10) If the radius of the merry-go-round is 3.25 feet, determine the linear velocity of the bug, in feet per minute.

10. (2.8) Throughout this problem, suppose $(3, -5)$ is on the terminal side of θ, when θ is drawn in standard position.

    a) Estimate the coordinates of a point on the terminal side of $\theta - 20^\circ$.
    b) Find the exact coordinates of a point on the terminal side of $\theta - 180^\circ$.
    c) Find the exact coordinates of a point on the terminal side of $90^\circ - \theta$.
    d) Find the exact coordinates of a point on the terminal side of $\theta + 720^\circ$.
    e) What quadrant is $180^\circ - \theta$ in?
    f) In terms of θ, what angle has $(3, 5)$ on its terminal side?

Solutions

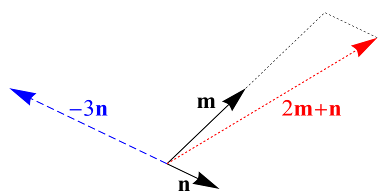
1. a) $\dfrac{2\pi}{3} = 2 \cdot \dfrac{\pi}{3} = 2 \cdot 60^\circ = \boxed{120^\circ}$.

   b) $-\dfrac{11\pi}{6} = -11 \cdot \dfrac{\pi}{6} = -11 \cdot 30^\circ = \boxed{-330^\circ}$.

   c) $\pi = \boxed{180^\circ}$.

   d) $\dfrac{7\pi}{2} = 7 \cdot \dfrac{\pi}{2} = 7 \cdot 90^\circ = \boxed{630^\circ}$.

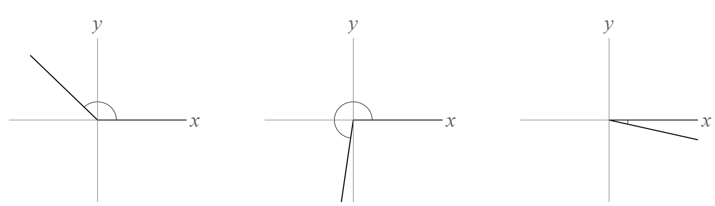
2. $-3\mathbf{n}$ is three times as long as $\mathbf{n}$, in the opposite direction. For $2\mathbf{m} + \mathbf{n}$, first double the length of $\mathbf{m}$ and then add $\mathbf{n}$ to the end via head-to-tail addition:



3. a) $\dfrac{3\pi}{4} = 3 \cdot \dfrac{\pi}{4} = 3 \cdot 45^\circ = 135^\circ$ is a diagonal angle in Quadrant II (see below at left).

   b) -13° is a very small clockwise angle in Quadrant IV (see below, in the middle).

   c) 262° is slightly less than 270°, so it is in Quadrant III near the y-axis (see below, at right).



4. To find a coterminal angle, add/subtract an appropriate multiple of 360°. Here, $800^\circ - 720^\circ = \boxed{80^\circ}$.

5. a) $2 \text{ golf } 3 = 2(3^2 + 2) = 2(11) = \boxed{22}$.

   b) $\text{golf}^{\,2}\, 1 = (\text{golf } 1)^2 = (1^2 + 2)^2 = 3^2 = \boxed{9}$.

   c) $\text{golf } 2 - 5 = (\text{golf } 2) - 5 = (2^2 + 2) - 5 = 6 - 5 = \boxed{1}$.

   d) $\text{golf } 2 \text{ golf } 5 = (\text{golf } 2)(\text{golf } 5) = (2^2 + 2)(5^2 + 2) = (6)(27) = \boxed{162}$.

   e) $\text{golf } 2 \cdot 5 = \text{golf } (2 \cdot 5) = \text{golf } 10 = 10^2 + 2 = \boxed{102}$.

6. a) The substitution of alpha $x = 2$ is shown in red below:

$$\begin{aligned} 3\,\text{alpha}^2 x + \text{ bravo}^2 x &= 30 \\ 3(2)^2 + \text{ bravo}^2 x &= 30 \\ 3(4) + \text{ bravo}^2 x &= 30 \\ 12 + \text{ bravo}^2 x &= 30 \\ \text{bravo}^2 x &= 18 \\ (\text{bravo } x)^2 &= 18 \\ \text{bravo } x &= \boxed{\pm\sqrt{18}} = \boxed{\pm 4.24264}. \end{aligned}$$

b) Combine the like terms, then isolate $\cos x$:

$$\begin{aligned} 12^2 &= 5^2 + 9^2 - 2(5)9\cos x \\ 144 &= 25 + 81 - 90\cos x \\ 144 &= 106 - 90\cos x \\ 38 &= -90\cos x \\ \boxed{\frac{38}{-90}} &= \cos x \\ \boxed{-.4222} &= \cos x \end{aligned}$$

c) This is a right triangle, so we can use the Pythagorean Theorem:

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 11.7^2 + 23.5^2 &= x^2 \\ 136.89 + 552.55 &= x^2 \\ 689.14 &= x^2 \\ \sqrt{689.14} &= x \\ \boxed{26.2515} &= x \end{aligned}$$

7. a) The angles of any triangle must sum to $180°$, so we have

$$\begin{aligned} (2x + 36°) + (3x + 5°) + (5x - 21°) &= 180° \\ 10x + 20° &= 180° \\ 10x &= 160° \\ x &= 16° \end{aligned}$$

Having found x, the three angles are

$$\begin{aligned} \angle A &= 2x + 36° = 2(16°) + 36° = \boxed{68°} \\ \angle B &= 3x + 5° = 3(16°) + 5° = \boxed{53°} \\ \angle C &= 5x - 21° = 5(16°) - 21° = \boxed{59°}. \end{aligned}$$

b) Since the three angles of $\triangle ABC$ all have different measure, the three sides have different lengths, so the triangle is $\boxed{\text{SCALENE}}$.

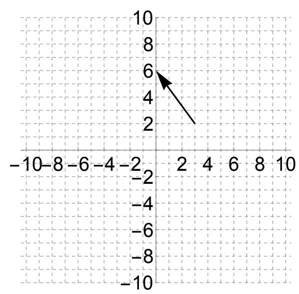
c) Since all three angles of $\triangle ABC$ have measure less than $90°$, the triangle is $\boxed{\text{ACUTE}}$.

8. a) $3\mathbf{v} + 2\mathbf{w} = 3\langle 5, -2\rangle + 2\langle -3, 4\rangle = \langle 15, -6\rangle + \langle -6, 8\rangle = \boxed{\langle 9, 2\rangle}$.

b) $|\mathbf{w}| = \sqrt{(-3)^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = \boxed{5}$.

c) $\mathbf{v} \cdot \mathbf{w} = 5(-3) + (-2)4 = -15 - 8 = \boxed{-23}$.

d) This vector should point 3 units left and 4 units up, so if it starts at $(3, 2)$, it must end at $(0, 6)$:



9. a) First, the angle needs to be converted to radians:

$$\theta = 40° \cdot \frac{\pi}{180°} = .698132$$

Then, use the formula for the area of a sector:

$$A = \frac{1}{2}r^2\theta = \frac{1}{2}(5.2 \text{ ft})^2(.698132) = \boxed{9.43874 \text{ ft}^2}.$$

b) $1733° \cdot \dfrac{\pi}{180°} = \boxed{30.2466}$.

c) i. We are given the angular velocity $\omega = 15$ rev/min. Since we are given an angle in degrees, we need to convert this to degrees per minute:

$$\omega = \frac{15 \text{ rev}}{\text{min}} \cdot \frac{360°}{1 \text{ rev}} = \frac{5400°}{\text{min}}.$$

Now, by definition of angular velocity, we have

$$\begin{aligned} \omega &= \frac{\theta}{t} \\ 5400°\text{min} &= \frac{800°}{t} \\ 5400°/\text{min}(t) &= 800° \\ t &= \frac{800°}{5400°/\text{ min}} = \boxed{.148148 \text{ min}}. \end{aligned}$$

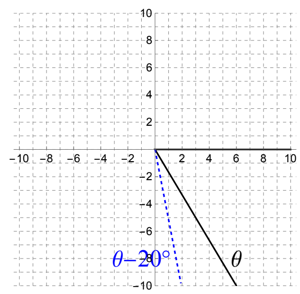
ii. The linear velocity is $v = r\omega$, where ω is in radians per unit of time. So we need to first convert ω to the correct units:

$$\omega = \frac{15 \text{ rev}}{\text{min}} \cdot \frac{2\pi}{1 \text{ rev}} = \frac{94.2478 \text{ rad}}{\text{min}}.$$

Then

$$v = r\omega = (3.25 \text{ ft})(94.2478 \text{ rad/min}) = \boxed{306.305 \text{ ft/min}}.$$

10. a) Start by drawing θ: on my picture below, θ is the solid black angle in Quadrant IV. To get from θ to $\theta - 20°$, you have to rotate another $20°$ clockwise from θ. That produces the blue dashed line in my picture:



From this picture, I estimate that a point on $\theta - 20°$ is $\boxed{(1, -5)}$ (although any point in Quadrant IV near the y-axis is reasonable).

b) To get from θ to $\theta - 180°$, we go in the opposite direction from θ, which multiplies both x and y by -1. This gives the point $\boxed{(-3, 5)}$ on $\theta - 180°$.

c) To get from θ to $90° - \theta$, we reflect across the diagonal line $y = x$ which interchanges x and y. A point on $90° - \theta$ is therefore $\boxed{(-5, 3)}$.

d) $\theta + 720°$ is coterminal to θ, so the same point is on the terminal side of $\theta + 720°$: $\boxed{(3, -5)}$.

e) To get from θ (which is in Quadrant IV) to $180° - \theta$, we reflect across the y-axis. This puts $180° - \theta$ in $\boxed{\text{Quadrant III}}$.

f) We get from θ to the desired angle in Quadrant I by reflecting across the x-axis. Thus the angle we want is $\boxed{360° - \theta}$.

1.3 Fall 2022 Exam 1

1. (2.9) $\boxed{\text{NC}}$ Convert each angle from radians to degrees:

   a) $\dfrac{-5\pi}{3}$ b) $\dfrac{7\pi}{6}$ c) 3π d) $-\dfrac{5\pi}{2}$

2. (2.8) $\boxed{\text{NC}}$ Sketch each angle in standard position.

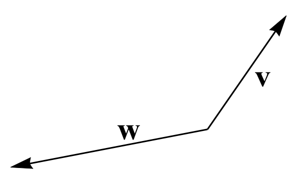
   a) $\dfrac{5\pi}{4}$ b) $110°$ c) $-270°$ d) $380°$

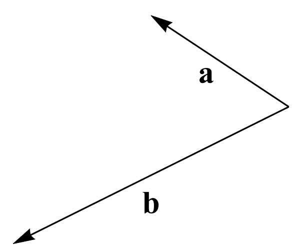
3. (2.9) $\boxed{\text{NC}}$ Compute the exact value of x, if the point $\left(x, \frac{1}{2}\right)$ is on the unit circle and lies in Quadrant I.

4. (1.3) $\boxed{\text{NC}}$ Let "grill" be the function defined by grill $x = 2 + 3x$. Compute each quantity:

   a) grill $2 + 3$
   b) grill $(2 + 3)$
   c) grill $3 \cdot 2$
   d) grill $(3 \cdot 2)$
   e) grill 3 grill 2

5. a) (2.4) $\boxed{\text{NC}}$ Vectors $\mathbf{v}$ and $\mathbf{w}$ are shown on the picture below at left. Sketch $\mathbf{v} + \mathbf{w}$ on the same picture.

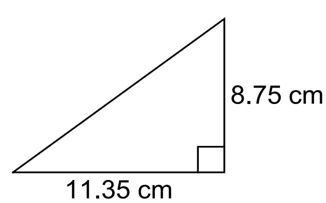




   b) (2.4) $\boxed{\text{NC}}$ Vectors $\mathbf{a}$ and $\mathbf{b}$ are shown on the picture above at right. Sketch $2\mathbf{a} - \frac{1}{2}\mathbf{b}$ on the same picture.

6. Parts (a), (b) and (c) of this question are not related to one another.

   a) (2.7) Compute the length of the hypotenuse of the triangle pictured below:



   b) (2.9) Convert $1752°$ to radians.

c) (2.10) Compute the area of a sector taken from a circle of radius 14 in, if the central angle of the sector is $72.5°$.

7. (1.2) Parts (a) and (b) of this question are not related to one another.

a) Solve for $\sin\theta$:
$$\frac{\sin\theta}{13.75} = \frac{.572}{9.5}$$

b) Solve for $\cos\theta$:
$$8^2 = 7^2 + 3^2 - 2(7)(3)\cos\theta$$

8. Answer each of these unrelated questions.

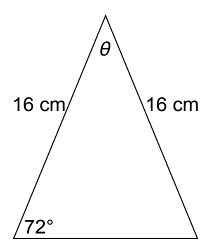
a) (2.5) If θ measures $142°$, what kind of angle is θ? (Choose from the words "acute", "right", "obtuse", "straight" or "reflex").

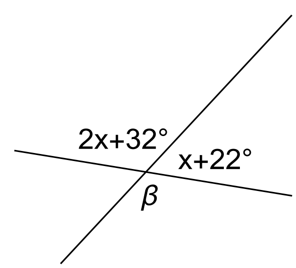
b) (2.7) Sketch a picture of a right isosceles triangle.

c) (2.8) Find an angle between $0°$ and $360°$ which is coterminal with $1005°$.

9. Parts (a), (b) and (c) of this question are not related to one another.

a) (2.6) Compute θ, where θ is in the picture below at left:





b) (2.6) Compute β ("beta"), where β is as in the picture above at right:

c) (2.6) An angle has measure equal to twice its complement. What is the measure of the angle?

10. Throughout this problem, let $\mathbf{v} = \langle 13, 17\rangle$ and let $\mathbf{w} = \langle 11, -4\rangle$.

a) (2.7) Compute the magnitude of $\mathbf{w}$.

b) (2.4) Compute $5\mathbf{v} - 4\mathbf{w}$.

c) (2.4) Compute $\mathbf{v} \cdot 3\mathbf{w}$.

Solutions

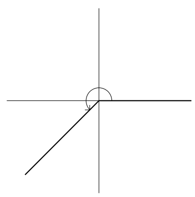
1. a) $\frac{-5\pi}{3} = -5 \cdot \frac{\pi}{3} = -5 \cdot 60° = \boxed{-300°}$.

   b) $\frac{7\pi}{6} = 7 \cdot \frac{\pi}{6} = 7 \cdot 30° = \boxed{210°}$.

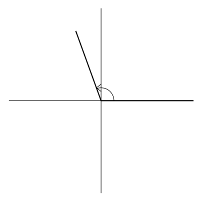
   c) $3\pi = 3 \cdot 180° = \boxed{540°}$.

   d) $-\frac{5\pi}{2} = 5 \cdot \frac{\pi}{2} = 5 \cdot 90° = \boxed{450°}$.

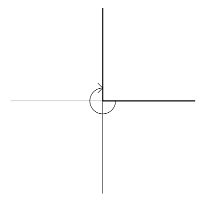
2. a)



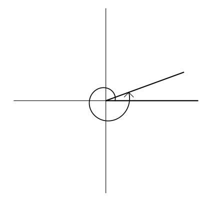
   b)



   c)



   d)



3. Start with the equation for the unit circle and plug in $y = \frac{1}{2}$:

$$\begin{aligned} x^2 + y^2 &= 1 \\ x^2 + \left(\frac{1}{2}\right)^2 &= 1 \\ x^2 + \frac{1}{4} &= 1 \\ x^2 &= \frac{3}{4} \\ x &= \pm\sqrt{\frac{3}{4}} \end{aligned}$$

   Since x is in Quadrant I, $x > 0$, so $x = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \boxed{\frac{\sqrt{3}}{2}}$.

4. a) grill $2 + 3$ = (grill 2) $+ 3 = (2 + 3 \cdot 2) + 3 = 8 + 3 = \boxed{11}$.

   b) grill $(2 + 3) =$ grill $5 = 2 + 3 \cdot 5 = \boxed{17}$.

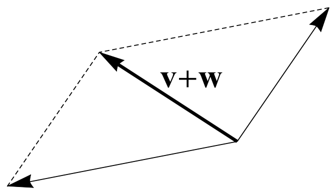
   c) grill $3 \cdot 2 =$ grill $(3 \cdot 2) =$ grill $6 = 2 + 3 \cdot 6 = \boxed{20}$.

   d) grill $(3 \cdot 2) =$ grill $6 = 2 + 3 \cdot 6 = \boxed{20}$.

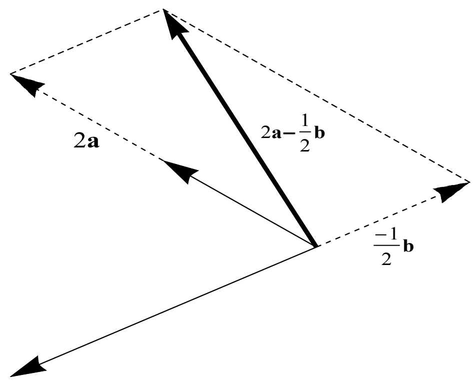
   e) grill 3 grill $2 =$ (grill 3)(grill 2) $= (2 + 3 \cdot 3)(2 + 3 \cdot 2) = (11)(8) = \boxed{88}$.

5.

(a)



(b)



6. a) Denote the hypotenuse length by c, and use the Pythagorean Theorem:

$$\begin{aligned} 11.35^2 + 8.75^2 &= c^2 \\ 128.823 + 76.5625 &= c^2 \\ 205.385 &= c^2 \\ \sqrt{205.385} &= c \\ \boxed{14.331} &= c. \end{aligned}$$

b) $1752° \cdot \frac{\pi}{180°} = \boxed{30.5782}$ radians.

c) First, convert the central angle to radians to get $\theta = 72.5° \cdot \frac{\pi}{180°} = 1.26536$ radians. Now, use the formula for the area of a sector:

$$A = \frac{1}{2}r^2\theta = \frac{1}{2}(14^2)(1.26536) = \boxed{124.005 \text{ sq in}}.$$

7. a) Cross-multiply this proportion to get

$$\begin{aligned} 9.5\sin\theta &= 13.75(.572) \\ \sin\theta &= \frac{13.75(.572)}{9.5} = \boxed{.827895}. \end{aligned}$$

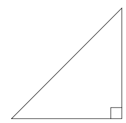
b) Combine the like terms and then isolate $\cos\theta$:

$$\begin{aligned} 8^2 &= 7^2 + 3^2 - 2(7)(3)\cos\theta \\ 64 &= 49 + 9 - 42\cos\theta \\ 64 &= 58 - 42\cos\theta \\ 6 &= -42\cos\theta \\ \boxed{-\frac{1}{7}} &= \cos\theta \end{aligned}$$

P.S. A decimal approximation of this is $-.142857$.

8. a) A $142°$ angle is $\boxed{\textbf{obtuse}}$, since it is between $90°$ and $180°$.

   b) The triangle should be a right triangle where the two legs are of the same length:



   c) $1005° - 2 \cdot 360° = 1005° - 720° = \boxed{285°}$.

9. a) Since the triangle is isosceles, the two angles at the bottom of the triangle are the same, and since the sum of the measures of the angles in a triangle is $180°$, we have

$$72° + 72° + \theta = 180°.$$

   Solve for θ to get $\theta = \boxed{36°}$.

   b) The two angles $2x + 32°$ and $x + 22°$ form a straight line, so

$$(2x + 32°) + (x + 22°) = 180°.$$

   Combine like terms to get $3x + 54° = 180°$, i.e. $3x = 126°$, and then solve for x to get $x = 42°$. Finally, $\beta = 2x + 32°$ (vertical angles have the same measure), so $\beta = 2(42°) + 32° = \boxed{116°}$.

   c) Let x be the angle, so that the complement of the angle is $90° - x$. The given information therefore translates as $x = 2(90° - x)$, i.e. $x = 180° - 2x$, i.e. $3x = 180°$, i.e. $x = \boxed{60°}$.

10. a) $|\mathbf{w}| = \sqrt{a^2 + b^2} = \sqrt{11^2 + (-4)^2} = \sqrt{137} = \boxed{11.705}$.

   b) $5\mathbf{v} - 4\mathbf{w} = 5\langle 13, 17\rangle - 4\langle 11, -4\rangle = \langle 65, 85\rangle - \langle 44, -16\rangle = \boxed{\langle 21, 101\rangle}$.

   c) $\mathbf{v} \cdot 3\mathbf{w} = \langle 13, 17\rangle \cdot \langle 33, -12\rangle = 13(33) + 17(-12) = 429 - 204 = \boxed{225}$.

1.4 Fall 2019 Exam 1

1. (2.9) $\boxed{\text{NC}}$ For each given angle, convert the angle from radians to degrees, and sketch the angle in standard position.

   a) $\frac{4\pi}{3}$ b) $\frac{\pi}{4}$ c) 4π d) $-\frac{5\pi}{6}$

2. (1.1) $\boxed{\text{NC}}$ Classify the following statements as true or false (circle your answer).

   a) $\sqrt{a^2+b^2} = a+b$.
   b) $\frac{1}{a-b} = \frac{1}{a} - \frac{1}{b}$.
   c) $(a+b)^2 = a^2+b^2$.

3. (1.3) $\boxed{\text{NC}}$ Let "soup" be the function defined by soup $x = 3 + \frac{1}{2}x$. Compute each quantity:

   a) soup $8 -$ soup 4
   b) soup $8 - 4$
   c) soup $(2 \cdot 4)$
   d) soup $2 \cdot 4$
   e) 2 soup 4

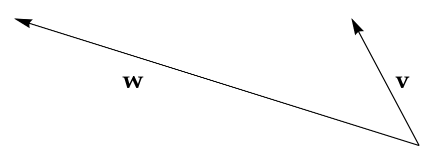
4. (2.5, 2.6) $\boxed{\text{NC}}$ Match each word to the letter of the phrase that most closely conveys its meaning (the choices start below, and extend on to the next page).

______	quadrantal	______	isosceles
______	supplementary	______	complementary
______	transversal	______	vertical
______	scalene	______	obtuse
______	terminal	______	coterminal

   A. the side of an angle where it starts
   B. the side of an angle where it ends
   C. a line that cuts across a pair of parallel lines
   D. two angles which add to $90°$
   E. two angles which add to $180°$
   F. two angles which add to $360°$
   G. two angles which differ by a multiple of $90°$
   H. two angles which differ by a multiple of $360°$
   I. a triangle with two sides of the same length
   J. a triangle with all three sides having the same length
   K. a triangle with no sides having the same length
   L. an angle measuring less than $90°$
   M. an angle measuring between $90°$ and $180°$

N. an angle measuring $180°$
O. an angle measuring more than $180°$
P. an angle which is a multiple of $90°$
Q. two angles which are across from one another, and therefore are equal

5. (2.4) NC Let **v** and **w** be the vectors indicated in the picture below.
   a) Sketch the vector $-\frac{1}{2}\mathbf{v}$ on the picture; label that vector "$-\frac{1}{2}\mathbf{v}$".
   b) Sketch the vector $\mathbf{v}+\mathbf{w}$ on the picture; label that vector "$\mathbf{v}+\mathbf{w}$".
   c) Sketch the vector $2\mathbf{v}-\mathbf{w}$ on the picture; label that vector "$2\mathbf{v}-\mathbf{w}$".

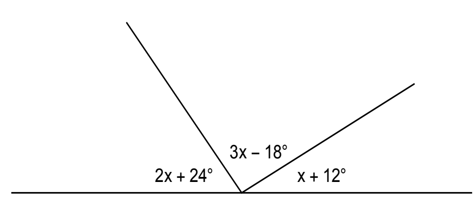


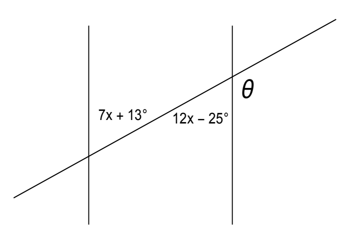
6. a) (2.7) Suppose that the hypotenuse of a right triangle has length 18.32 inches. If one leg of the triangle measures 13.55 inches, how long is the other leg?
   b) (2.10) You are judging a pumpkin pie cooking contest. Contestant K gives you a slice which is a $22°$ wedge taken from a pie of diameter 14 inches. Contestant L gives you a slice which is a $28°$ wedge taken from a pie of diameter 12 inches. Which contestant gave you a larger slice of pie? Explain your answer. (Assume that the heights of the pies are equal, and can therefore be ignored.)

7. a) (2.8) The point (x, y) is on the unit circle. If $x = -.3$, what are all possible values of y?
   b) (2.6) Two angles of a triangle measure $42°$ and $93°$. Find the measure of the third angle of the triangle.
   c) (2.9) Convert $283°$ to radians.
   d) (1.2) Solve for $\cos\theta$:

$$5^2 = 4^2 + (2.5)^2 + 2(4)(2.5)\cos\theta$$

8. a) (2.6) Find the value of x, in the picture below at left.
   b) (2.60 Assume that the two lines that appear in the picture below at right are in fact parallel. Find θ:





9. Throughout this problem, suppose $\mathbf{v} = \langle 11, -16 \rangle$ and $\mathbf{w} = \langle 13, 8 \rangle$.

   a) (2.4) Compute $2\mathbf{v} + 5\mathbf{w}$.
   b) (2.7) Compute the magnitude of $\mathbf{w}$.
   c) (2.4) Compute $\mathbf{v} \cdot \mathbf{w}$.

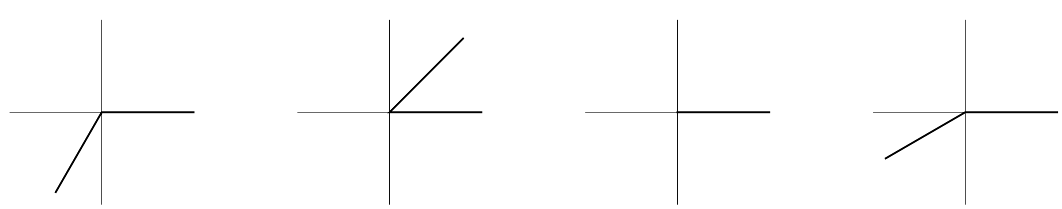
Solutions

1. a) $\frac{4\pi}{3} = 4 \cdot \frac{\pi}{3} = 4 \cdot 60° = \boxed{240°}$. This angle is in Quadrant III, as shown below at left.

   b) $\frac{\pi}{4} = \boxed{45°}$. This angle is in Quadrant I, as shown below in the second picture.

   c) $4\pi = 4 \cdot 180° = \boxed{720°}$. This angle is two full revolutions, so it begins and ends at the same place, as shown below in the third picture.

   d) $-\frac{5\pi}{6} = -5 \cdot \frac{\pi}{6} = -5 \cdot 30° = \boxed{-150°}$. This angle is in Quadrant III, as shown in the last picture below.



2. a) $\sqrt{a^2 + b^2} = a + b$ is **false** (try $a = 3$, $b = 4$, for example).

   b) $\frac{1}{a-b} = \frac{1}{a} - \frac{1}{b}$ is **false** (try $a = 2$, $b = 4$ for example).

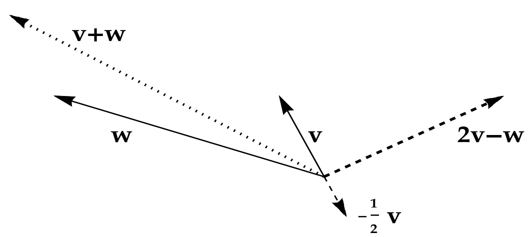
c) $(a+b)^2 = a^2 + b^2$ is **false** (you need to FOIL $(a+b)^2$).

3. a) soup $8 -$ soup $4 = (3 + \frac{1}{2}(8)) - (3 + \frac{1}{2}(4)) = 7 - 5 = \boxed{2}$.
   b) soup $8 - 4 =$ (soup $8) - 4 = (3 + \frac{1}{2}(8)) - 4 = 7 - 4 = \boxed{3}$.
   c) soup $(2 \cdot 4) =$ soup $8 = 3 + \frac{1}{2}(8) = \boxed{7}$.
   d) soup $2 \cdot 4 =$ soup $(2 \cdot 4) = \boxed{7}$, the same as part (c).
   e) 2 soup $4 = 2(3 + \frac{1}{2}(4)) = 2(5) = \boxed{10}$.

4. The correct answers are as follows:

P	quadrantal	**I**	isosceles
E	supplementary	**D**	complementary
C	transversal	**Q**	vertical
K	scalene	**M**	obtuse
B	terminal	**H**	coterminal

5. The vectors are as indicated below:



6. a) Use the Pythagorean Theorem (you could call the unknown side either a or b):

$$\begin{aligned} a^2 + b^2 &= c^2 \\ (13.55)^2 + b^2 &= 18.32^2 \\ 183.603 + b^2 &= 335.3622 \\ b^2 &= 152.019 \\ b &= \sqrt{152.019} \approx \boxed{12.3296 \text{ in}}. \end{aligned}$$

   b) First, convert the angles to radians so the area formula can be used: K's angle is $22° \cdot \frac{\pi}{180°} = .3839$ and L's angle is $28° \cdot \frac{\pi}{180°} = .4887$. Therefore, the area of K's slice is $A = \frac{1}{2}r^2\theta = \frac{1}{2}(7^2)(.3839) = 9.4073$ square inches (note that since the diameter is 14, the radius is 7), and the area of L's slice is $A = \frac{1}{2}r^2\theta = \frac{1}{2}(6)^2(.4887) = 8.7966$ square inches (again, note the radius is 6 since the diameter is 12). Since K's area is larger, we conclude that $\boxed{\textbf{K}}$ gave you the bigger slice.

7. a) Use the unit circle equation:

$$\begin{aligned} x^2 + y^2 &= 1 \\ (-.3)^2 + y^2 &= 1 \\ .09 + y^2 &= 1 \\ y^2 &= .91 \\ y &= \pm\sqrt{.91} \approx \boxed{\pm .9539}. \end{aligned}$$

b) The third angle is $180° - 42° - 93° = \boxed{45°}$.

c) $283° = 283° \cdot \frac{\pi}{180°} = \boxed{4.93928}$ radians.

d) First, square the numbers out and combine the like terms on the right to get

$$\begin{aligned} 5^2 &= 4^2 + (2.5)^2 + 2(4)(2.5)\cos\theta \\ 25 &= 16 + 6.25 + 20\cos\theta \\ 25 &= 22.25 + 20\cos\theta \end{aligned}$$

Next, subtract 22.25 from both sides (don't combine the terms on the right since they aren't like terms), and only at the end, divide by 20 to isolate $\cos\theta$. You will get $\cos\theta = \boxed{.1375}$.

8. a) The angles form a straight line, so they add to $180°$:

$$\begin{aligned} (2x + 24°) + (3x - 18°) + (x + 12°) &= 180° \\ 6x + 18° &= 180° \\ x &= \boxed{27°}. \end{aligned}$$

b) The two angles with xs in them are equal, so we can solve for x:

$$\begin{aligned} 7x + 13° &= 12x - 25° \\ 38° &= 5x \\ 7.6° &= x \end{aligned}$$

That means $12x - 25° = 12(7.6°) - 25° = 66.2°$, so $\theta = 180° - 66.2° = \boxed{113.8°}$.

9. a) $2\mathbf{v} + 5\mathbf{w} = 2\langle 11, -16\rangle + 5\langle 13, 8\rangle = \langle 22, -32\rangle + \langle 65, 40\rangle = \boxed{\langle 87, 8\rangle}$.

b) The magnitude is $|\mathbf{w}| = \sqrt{a^2 + b^2} = \sqrt{13^2 + 8^2} = \sqrt{169 + 64} = \boxed{\sqrt{233}} = \boxed{15.26}$.

c) $\mathbf{v} \cdot \mathbf{w} = 11(13) + (-16)8 = 143 - 128 = \boxed{15}$.

1.5 Fall 2018 Exam 1

1. (2.9) $\boxed{\text{NC}}$ Convert the following angles from radians to degrees:

   a) $\frac{2\pi}{3}$ b) $\frac{-5\pi}{4}$ c) π

2. $\boxed{\text{NC}}$ Classify the following statements as true or false (circle your answer):

   a) (2.8) Angles measuring $50°$ and $410°$ are coterminal.

   b) (2.6) Angles measuring $50°$ and $40°$ are supplementary.

   c) (2.5) If an angle measures $200°$, then it is an obtuse angle.

   d) (2.8) When drawn in standard position, an angle measuring $200°$ ends in Quadrant II.

3. (1.3) $\boxed{\text{NC}}$ Let "run" be the function defined by run $x = 5 + 3x$. Compute each quantity:

   a) run $1 + 2$

   b) run $(1 + 2)$

   c) run $3 \cdot 2$

   d) run $(3 \cdot 2)$

   e) 3 run 2

4. a) (2.9) Convert 8 radians to degrees.

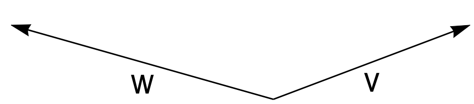
   b) (1.2) Solve for $\sin x$:

$$\frac{\sin x}{25.63} = \frac{.735}{18.51}$$

5. (2.4) $\boxed{\text{NC}}$ Let $\mathbf{v}$ and $\mathbf{w}$ be the vectors indicated in the picture below.

   a) Sketch the vector $-2\mathbf{v}$ on the picture; label that vector "$-2\mathbf{v}$".

   b) Sketch the vector $\mathbf{v} + \mathbf{w}$ on the picture; label that vector "$\mathbf{v} + \mathbf{w}$".



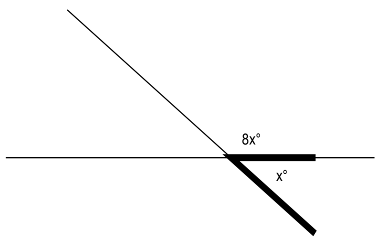
6. a) (2.9) The point (x, y) is on the unit circle. If $y = .5$, what are all possible values of x?

   b) (2.10) Find the length of an arc subtended from a circle of radius 22.6 inches, if the arc is subtended by an angle of $.81$ radians.

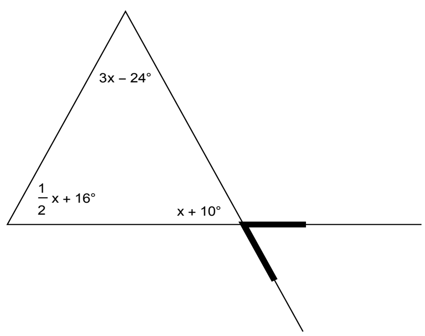
   c) (2.10) Suppose the area of a sector whose central angle is $147°$ has area 29 square feet. What is the radius of the circle from which this sector is taken?

7. (2.6) In each problem (a)-(c), find the measure of the angle which is drawn with thick lines:

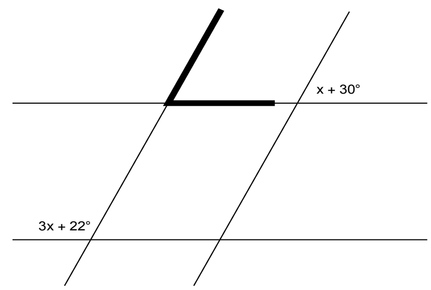
   a)



   b)



   c) In this picture, assume that there are two sets of parallel lines:



8. Throughout this problem, suppose $\mathbf{v} = \langle 3, -2 \rangle$ and $\mathbf{w} = \langle 5, 2 \rangle$.

   a) (2.4) Find $3\mathbf{v} - \mathbf{w}$.

   b) (2.7) Find the magnitude of $\mathbf{v}$.

   c) (2.4) Find $\mathbf{v} \cdot \mathbf{w}$.

Solutions

1. a) $\frac{2\pi}{3} = 2 \cdot \frac{\pi}{3} = 2 \cdot 60^\circ = \boxed{120^\circ}$

   b) $\frac{-5\pi}{4} = -5 \cdot \frac{\pi}{4} = -5 \cdot 45^\circ = \boxed{-225^\circ}$

   c) $\pi = \boxed{180^\circ}$

2. a) TRUE (the difference between these angles is a multiple of 360°).

   b) FALSE (the angles do not add to 180°).

   c) FALSE (obtuse angles are between 90° and 180°).

   d) FALSE (200° is in Quadrant III).

3. a) run $1 + 2 =$ run $(1) + 2 = (5 + 3(1)) + 2 = 8 + 2 = \boxed{10}$.

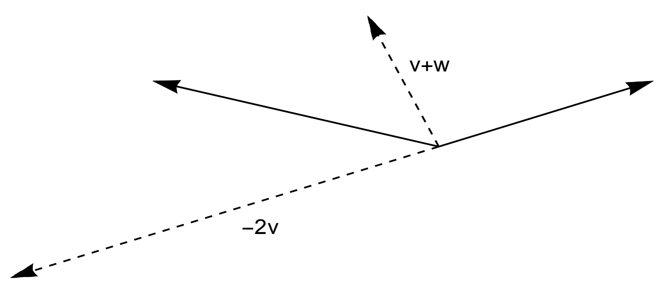
   b) run $(1 + 2) =$ run $3 = 5 + 3(3) = 5 + 9 = \boxed{14}$.

   c) run $3 \cdot 2 =$ run $6 = 5 + 3(6) = 5 + 18 = \boxed{21}$.

   d) run $(3 \cdot 2) =$ run $6 = 5 + 3(6) = 5 + 18 = \boxed{21}$.

   e) 3 run $2 = 3(5 + 3(2)) = 3(5 + 6) = 3(11) = \boxed{33}$.

4. See below:



5. a) $8 \cdot \frac{180^\circ}{\pi} = \boxed{458.37^\circ}$.

   b) Cross-multiply to get $18.51 \sin x = .735(25.63)$; then divide both sides by 18.51 to get $\sin x = \frac{.735(25.63)}{18.51} = \boxed{1.01772}$.

6. a) Since (x, y) is on the unit circle, $x^2 + y^2 = 1$. Substitute to obtain

$$\begin{aligned} x^2 + (.5)^2 &= 1 \\ x^2 + .25 &= 1 \\ x^2 &= .75 \\ x &= \pm\sqrt{.75} \approx \boxed{\pm .866}. \end{aligned}$$

   b) We have $s = r\theta = 22.6(.81) = \boxed{18.306 \text{ in}}$.

   c) Using the formula $A = \frac{1}{2}r^2\theta$, and substituting in $A = 29$ and $\theta = 147^\circ \cdot \frac{\pi}{180^\circ} = 2.566$ radians, we get $29 = \frac{1}{2}r^2(2.566)$. Multiply both sides by 2 and divide both sides by 2.566 to get $r^2 = 22.598$, i.e. $r = \sqrt{22.598} \approx \boxed{4.753 \text{ ft}}$.

7. a) Since the angles form a line, $8x + x = 180°$, i.e. $9x = 180°$, i.e. $x = \boxed{20°}$.

   b) The three angles of a triangle add to $180°$, so $x+10°+\frac{1}{2}x+16°+3x-24° = 180°$, i.e. $4.5x + 2° = 180°$, i.e. $4.5x = 178°$, i.e. $x = 39.555°$. The thick angle is $x + 10 = \boxed{49.555°}$.

   c) From the picture, angles $x + 30°$ and $3x + 22°$ are supplementary, so $x + 30° + 3x + 22° = 180°$, i.e. $4x + 52° = 180°$, i.e. $4x = 128°$, i.e. $x = 36°$. The thick angle is $x + 30 = \boxed{66°}$.

8. a) $3\mathbf{v} - \mathbf{w} = 3\langle 3, -2\rangle - \langle 5, 2\rangle = \langle 9, -6\rangle - \langle 5, 2\rangle = \boxed{\langle 4, -8\rangle}$.

   b) $|\mathbf{v}| = \sqrt{3^2 + (-2)^2} = \sqrt{9+4} = \boxed{\sqrt{13}} \approx \boxed{3.606}$.

   c) $\mathbf{v} \cdot \mathbf{w} = 3(5) + (-2)2 = 15 - 4 = \boxed{11}$.

1.6 Fall 2017 Exam 1

1. (2.9) $\boxed{\text{NC}}$ Convert the following angles from radians to degrees:

   a) $\frac{4\pi}{3}$ b) $\frac{3\pi}{2}$ c) $-\pi$

2. (2.8) $\boxed{\text{NC}}$ Draw the angle $250°$ in standard position.

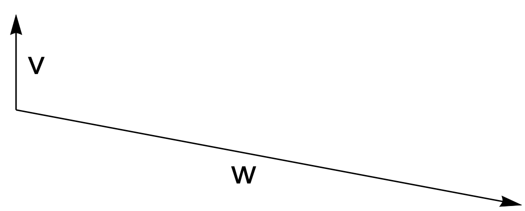
3. (1.3) $\boxed{\text{NC}}$ Let "sub" be the function defined by sub $x = 2x - 3$. Compute each quantity:

   a) sub $4 - 3$ b) sub $4 \cdot 3$ c) sub $(4) \cdot 3$ d) 3 sub 4

4. (2.4) $\boxed{\text{NC}}$ Let $\mathbf{v}$ and $\mathbf{w}$ be the vectors indicated in the picture below.

   a) Sketch the vector $-\frac{1}{2}\mathbf{w}$ on the picture; label that vector "$-\frac{1}{2}\mathbf{w}$".

   b) Sketch the vector $2\mathbf{v} + \mathbf{w}$ on the picture; label that vector "$2\mathbf{v} + \mathbf{w}$".



5. Parts (a), (b), and (c) of this question are not related to one another.

   a) (2.8) Find the measure of two different angles, one of which is less than $624°$ and one of which is greater than $624°$, both of which are coterminal with $624°$.

   b) (2.9) Convert 5.6 radians to degrees (write your answer as an exact answer or as a decimal, rounded to two or more decimal places).

   c) (1.2) Solve for x:
   $$\frac{x - 100}{3.25} = \frac{x}{4.8}$$

6. a) (2.9) The point (x, y) is on the unit circle. If $x = -.275$, what are all possible values of y?

   b) (2.10) Find the area of a sector whose central angle is $53°$, taken from a circle whose diameter is 6 feet.

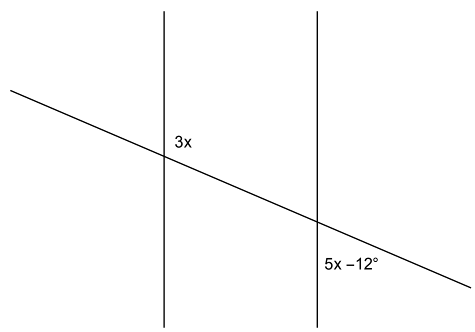
7. (2.10) A dragster has two front tires of radius 16 inches, and two rear tires of radius 26 inches. If the dragster is travelling with all four tires on the ground, with no skidding or friction, and rear tires rotate at 35 revolutions per second, what is the angular velocity of the front tires (in radians per second)?

8. (2.6) In each problem (a)-(c), find x:

   a) Two complementary angles have measures $x + 34°$ and $3x - 20°$.

   b) The three angles of a triangle are $x + 14°$, $3x - 35°$ and $7x + 3°$.

   c) x is as in the picture below, where the vertical lines are parallel:



9. a) (2.4) Suppose $\mathbf{u} = \langle -3, 8 \rangle$ and $\mathbf{v} = \langle 4, 5 \rangle$. Find $3\mathbf{u} + \mathbf{v}$.

   b) (2.4) Find $\mathbf{u} \cdot \mathbf{v}$, where $\mathbf{u}$ and $\mathbf{v}$ are as in part (a).

   c) (2.7) Find the magnitude of the vector $\mathbf{z} = \langle 6.3, 5.2 \rangle$.

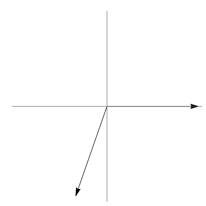
Solutions

1. a) $\frac{4\pi}{3} = 4 \cdot \frac{\pi}{3} = 4 \cdot 60^\circ = \boxed{240^\circ}$.

   b) $\frac{3\pi}{2} = 3 \cdot \frac{\pi}{2} = 3 \cdot 90^\circ = \boxed{270^\circ}$.

   c) $-\pi = \boxed{-180^\circ}$.

2. Since 250° is a little less than 270°, this angle should have its terminal side just west of south (in Quadrant III):



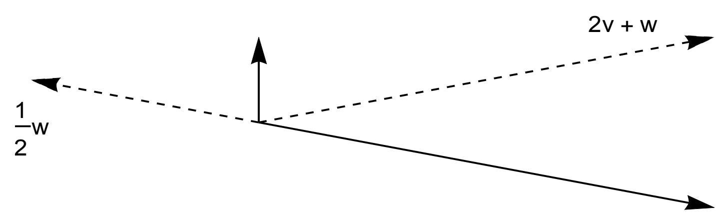
3. a) sub $4 - 3$ = sub$(4) - 3 = (2(4) - 3) - 3 = 5 - 3 = \boxed{2}$.

   b) sub $4 \cdot 3$ = sub $12 = 2(12) - 3 = 24 - 3 = \boxed{21}$.

   c) sub$(4) \cdot 3 = (2(4) - 3) \cdot 3 = 5 \cdot 3 = \boxed{15}$.

   d) 3 sub $4 = 3(2(4) - 3) = 3(5) = \boxed{15}$. (This is the same as (b).)

4. To get $-\frac{1}{2}\mathbf{w}$, sketch a vector in the opposite direction as $\mathbf{w}$, but half as long. To get $2\mathbf{v} + \mathbf{w}$, first draw $2\mathbf{v}$ (twice as long as $\mathbf{v}$, in the same direction) and then add by drawing a parallelogram. You end up with the following:



5. a) For a smaller angle, subtract 360° from 624° to get $\boxed{264^\circ}$; for a larger angle, add 360° to 624° to get $\boxed{984^\circ}$.

   b) Multiply by $\frac{180^\circ}{\pi}$ to get $\boxed{320.85^\circ}$.

   c) Cross-multiply to get $4.8(x - 100) = 3.25x$. Distribute on the left-hand side to get $4.8x - 480 = 3.25x$; combine the like terms to get $1.55x = 480$; last, divide by 1.55 to get $x = \boxed{309.677}$.

6. a) From the unit circle formula $x^2 + y^2 = 1$, we see $x^2 + (-.275)^2 = 1$. Squaring out, this gives $x^2 + .0756 = 1$, i.e. $x^2 = 1 - .0756 = .9243$. Take the square root of both sides to get $x = \pm\sqrt{.9243} = \boxed{\pm .9614}$.

   b) The radius is half of the given diameter, i.e. $r = 3$. Convert the angle to radians by multiplying by $\frac{\pi}{180^\circ}$ to get $\theta = .925$ radians. Then $A = \frac{1}{2}r^2\theta = \frac{1}{2}(3)^2(.925) = \boxed{4.1625 \text{ sq ft}}$.

7. The rear tires have angular velocity $\omega_{rear} = 35(2\pi) = 219.911$ rad/sec, so their linear velocity is $v_{rear} = r_{rear}\omega_{rear} = 26(219.911) = 5717.1$ in/sec. The front tires have the same linear velocity as the back tires, so $v_{front} = 5717.1$ in/sec as well. Finally, $\omega_{front} = \frac{v_{front}}{r_{front}} = \frac{5717.1}{16} = \boxed{357.356 \text{ radians per second}}$.

8. a) The angles are complementary, so $(x + 34°) + (3x - 20°) = 90°$. Combine the like terms to get $4x + 14° = 90°$, i.e. $4x = 76°$, i.e. $x = \boxed{19°}$.

   b) The angles sum to $180°$: $(x + 14°) + (3x - 35°) + (7x + 3°) = 180°$. Combine the like terms to get $11x - 18° = 180°$, i.e. $11x = 198°$, i.e. $x = \boxed{18°}$.

   c) The angles are supplementary, so $3x + 5x - 12° = 180°$. This means $8x = 192°$, i.e. $x = \boxed{24°}$.

9. a) $3\mathbf{u} + \mathbf{v} = \langle -9, 24\rangle + \langle 4, 5\rangle = \boxed{\langle -5, 29\rangle}$.

   b) $\mathbf{u} \cdot \mathbf{v} = (-3)4 + 8(5) = \boxed{28}$.

   c) $|\mathbf{z}| = \sqrt{6.3^2 + 5.2^2} = \sqrt{66.73} = \boxed{8.16884}$.

1.7 Fall 2016 Exam 1

1. (2.9) $\boxed{\text{NC}}$ Convert the following angles from radians to degrees, and draw them in standard position:

   a) $\frac{5\pi}{6}$ b) $\frac{\pi}{2}$ c) $\frac{8\pi}{3}$ d) $\frac{-4\pi}{4}$

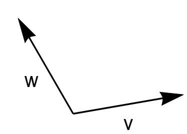
2. (1.3) $\boxed{\text{NC}}$ Let cat be the function defined by cat $x = x^2 + 1$. Compute each quantity:

   a) cat $2 + 3$ b) cat $2 \cdot 3$ c) cat(2) + 3 d) 2 cat 3

3. (2.4) $\boxed{\text{NC}}$ Let $\mathbf{v}$ and $\mathbf{w}$ be the vectors indicated in the picture below.

   a) Sketch the vector $2\mathbf{v} + \mathbf{w}$ on the picture; label that vector "$2\mathbf{v} + \mathbf{w}$".

   b) Sketch the vector $\mathbf{w} - \mathbf{v}$ on the picture; label that vector "$\mathbf{w} - \mathbf{v}$".



4. a) (2.8) Find an angle measuring between $0°$ and $360°$ which is coterminal with $4375°$.

   b) (2.9) Convert $222°$ to radians (write your answer as a decimal, rounded to two or more decimal places).

   c) (2.9) Find the coordinates of a point on the unit circle which is on the terminal side of a $540°$ angle, drawn in standard position.

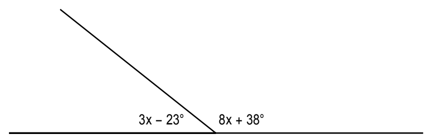
   d) (2.6) Draw a picture of a triangle which is isoceles, but not equilateral.

5. a) (2.6) An angle measures $18°$ more than its complement. What is the measure of the angle?

   b) (2.7) The three angles of a triangle measure x, $x + 25°$ and $x - 40°$. Is this triangle an acute triangle, a right triangle, or an obtuse triangle?

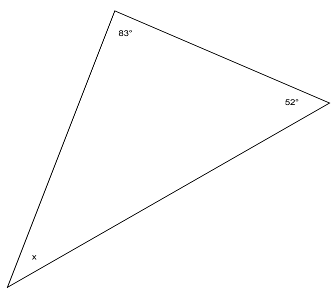
6. a) (2.10) A Ferris wheel makes $.065$ revolutions in a minute. If the radius of the Ferris wheel is 120 feet, what is the linear velocity of someone sitting in a bucket on the edge of the Ferris wheel?

   b) (2.6) Find the measure of each angle in this picture:

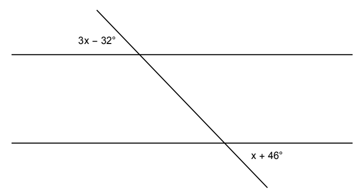


7. (2.6) In each picture, find x (in (b), assume the lines that look horizontal are parallel):

a)



b)



8. Throughout this question, assume that $\mathbf{u} = \langle -5, 3\rangle$, and $\mathbf{v} = \langle -1, 8\rangle$.

   a) (2.4) Compute $2\mathbf{u} + \mathbf{v}$.

   b) (2.4) Compute $\mathbf{u} \cdot \mathbf{v}$.

   c) (2.7) Compute $|\mathbf{u} + \mathbf{v}|$.

Solutions

1. a) $\frac{5\pi}{6} = 5 \cdot \frac{\pi}{6} = 5 \cdot 30° = \boxed{150°}$. In standard position, this angle should point just north of the negative x-axis.

b) $\frac{\pi}{2} = \boxed{90^\circ}$. In standard position, this angle points due north.

c) $\frac{8\pi}{3} = 8 \cdot \frac{\pi}{3} = 8 \cdot 60^\circ = \boxed{480^\circ}$. In standard position, this angle goes all the way around once, then halfway around again to end up on the negative x-axis.

d) $\frac{-4\pi}{4} = -\pi = \boxed{-180^\circ}$. In standard position, this angle is on the negative x-axis.

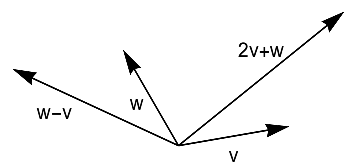
2. a) cat $2 + 3 =$ cat$(2) + 3 = (2^2 + 1) + 3 = 5 + 3 = \boxed{8}$.

   b) cat $2 \cdot 3 =$ cat $6 = 6^2 + 1 = \boxed{37}$.

   c) cat$(2) + 3 = (2^2 + 1) + 3 = 5 + 3 = \boxed{8}$.

   d) 2 cat $3 = 2(3^2 + 1) = 2(10) = \boxed{20}$.

3.



4. a) $4375 \div 360 = 12.15$ so the angle is $4375^\circ - 12 \cdot 360^\circ = \boxed{55^\circ}$.

   b) $222^\circ \times \frac{\pi}{180^\circ} = \boxed{3.87}$ radians.

   c) $540^\circ = 360^\circ + 180^\circ$ so the terminal side of this angle points on the negative x-axis. The point on the negative x-axis which is on the unit circle is $\boxed{(-1, 0)}$.

   d) The triangle should have two sides that are the same length, but not all three sides of the same length.

5. a) Let x be the angle; we have $x = 18^\circ + (90^\circ - x)$. Solve for x to get $x = \boxed{54^\circ}$.

   b) The three angles sum to 180°, so $x + (x + 25^\circ) + (x - 40^\circ) = 180^\circ$. Solve for x to get $x = 65^\circ$. This makes the three angles $x = 65^\circ$, $x + 25^\circ = 90^\circ$ and $x - 40^\circ = 25^\circ$. Since one of the angles is 90°, the triangle is a $\boxed{\textbf{right}}$ triangle.

6. a) The angular velocity is $\omega = .065 \cdot 2\pi = .408$ radians per minute. The linear velocity is therefore $v = r\omega = 120(.408) \approx \boxed{\text{49 feet per minute}}$.

   b) The angles are supplementary, so $(3x - 23^\circ) + (8x + 38^\circ) = 180^\circ$. Solve for x to get $x = 15^\circ$; the angles are therefore $3(15^\circ) - 23^\circ = \boxed{22^\circ}$ and $8(15^\circ) + 38^\circ = \boxed{158^\circ}$.

7. a) The angles sum to 180°: $x + 52^\circ + 83^\circ = 180^\circ$. Solve for x to get $\boxed{x = 45^\circ}$.

   b) The angles are equal, so $3x - 32^\circ = x + 48^\circ$. Solve for x to get $\boxed{x = 39^\circ}$.

8. a) $2\mathbf{u} + \mathbf{v} = \langle -10, 6 \rangle + \langle -1, 8 \rangle = \boxed{\langle -11, 14 \rangle}$.

b) $\mathbf{u} \cdot \mathbf{v} = (-5)(-1) + 3(8) = 5 + 24 = \boxed{29}$.

c) $|\mathbf{u} + \mathbf{v}| = |\langle -6, 11 \rangle| = \sqrt{(-6)^2 + 11^2} = \sqrt{36 + 121} = \boxed{\sqrt{157}} \approx \boxed{12.53}$.