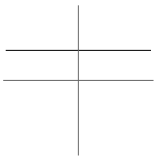
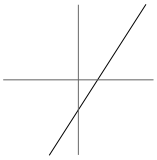
I. Linear and absolute value functions

Constant functions: $f(x) = c$



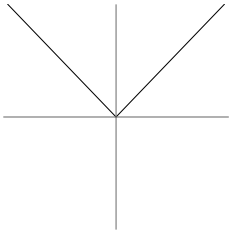
Graph: horizontal line at height c
Symmetry: even

Linear functions: $f(x) = mx + b$ (where $m \neq 0$)



Graph: line with slope m and y-intercept $(0, b)$
Domain: all real numbers
Range: all real numbers
Symmetry: odd if $b = 0$, otherwise none
one-to-one? Yes
Inverse function: $f^{-1}(x) = \frac{x-b}{m}$

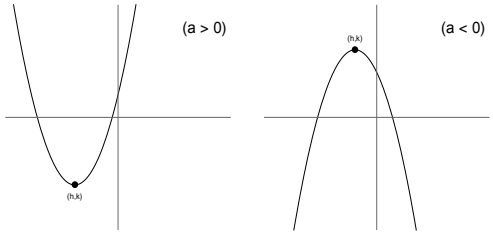
Absolute value function: $f(x) = |x|$



Graph: piecewise linear V shape with corner at origin
Domain: all real numbers
Range: $[0, \infty)$
Symmetry: even
one-to-one? No
Partial inverses: $\pm x$

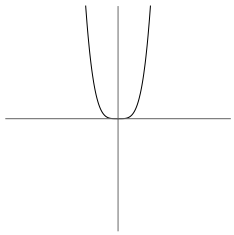
II. Polynomials

Quadratic functions: $f(x) = ax^2 + bx + c$ (where $a \neq 0$)



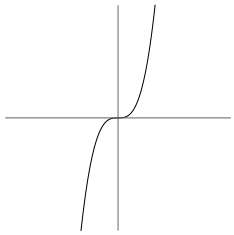
Vertex: (h, k) where $h = \frac{-b}{2a}$ = average of the x-ints; $k = f(h)$
Graph: parabola with vertex at (h, k); opens upward if $a > 0$ and opens downward if $a < 0$
Domain: all real numbers
Range: $[k, \infty)$ if $a > 0$; $(-\infty, k]$ if $a < 0$
Symmetry: about line $x = h$
one-to-one? No

Even power functions: $f(x) = x^n$ (where $n \geq 2$ is an even whole number)



Graph: looks mostly like the graph of x^2
Domain: all real numbers
Range: $[0, \infty)$
Symmetry: even
one-to-one? No
Partial inverses: $\pm \sqrt[n]{x}$

Odd power functions: $f(x) = x^n$ (where $n \geq 3$ is an odd whole number)



Graph: looks mostly like the graph of x^3
Domain: all real numbers
Range: all real numbers
Symmetry: odd
one-to-one? Yes
Inverse: $f^{-1}(x) = \sqrt[n]{x}$

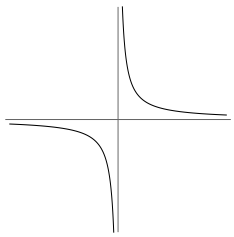
General polynomials: $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$ (with $a_n \neq 0$)

Degree: n (the highest power of x that occurs)
Leading coefficient (LC): a_n (the number on the highest power of x)
Graph: continuous, smooth curve with tails pointing upward/downward
Domain: all real numbers
Tail behavior:
- if degree is even and LC is positive, both tails point up
- if degree is even and LC is negative, both tails point down
- if degree is odd and LC is positive, right tail points up and left tail points down
- if degree is odd and LC is negative, right tail points down and left tail points up

Asymptotes: none
x-ints: set $f(x) = 0$ and factor; when graphing, take multiplicity into account at each x-int

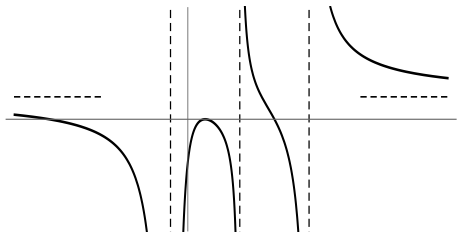
III. Rational functions

Reciprocal function: $f(x) = \frac{1}{x} = x^{-1}$



Graph: hyperbola (two pieces, one in Quadrant I and one in Quadrant III)
Domain: all real numbers except 0
Range: all real numbers except 0
HA: $y = 0$
VA: $x = 0$
one-to-one? Yes
Inverse function: $f^{-1}(x) = \frac{1}{x}$ (this function is its own inverse)

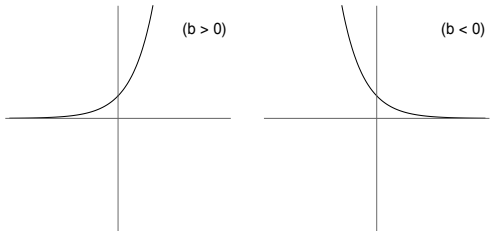
Rational functions: $f(x) = \frac{\text{polynomial}}{\text{polynomial}}$



Graph: generally consists of multiple smooth pieces, merging toward asymptotes; a typical graph looks like the picture at left
HA: let x be large (or compare highest powers on top and bottom)
VA: set bottom $= 0$ and check if top $\neq 0$

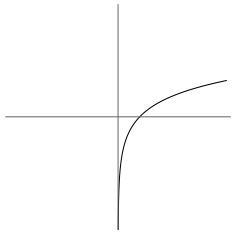
IV. Exponential and logarithmic functions

Exponential functions: $f(x) = b^x$ (especially $f(x) = e^x$)



Graph: increases from left to right if $b > 1$, decreases from left to right if $0 < b < 1$; goes through $(0, 1)$ and $(1, b)$
Domain: all real numbers
Range: $(0, \infty)$
HA: $y = 0$ (one side only)
VA: none
one-to-one? Yes
Inverse function: $f^{-1}(x) = \log_b x$

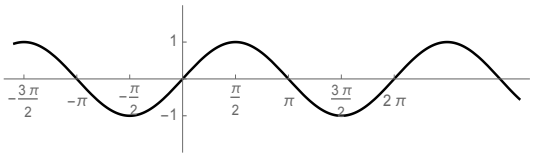
Logarithmic functions: $f(x) = \log_b x$ (especially $f(x) = \ln x$)



Graph: increases from left to right if $b > 1$; goes through $(1, 0)$ and $(b, 1)$
Domain: $(0, \infty)$
Range: all real numbers
HA: none
VA: $x = 0$ (one side only)
one-to-one? Yes
Inverse function: $f^{-1}(x) = b^x$

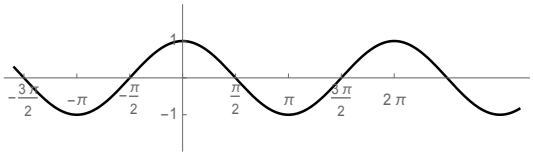
V. Trigonometric and inverse trigonometric functions

Sine function: $f(x) = \sin x$



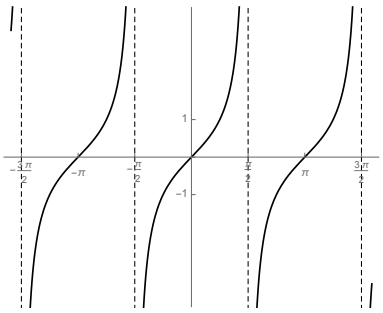
Graph: wave passing through $(0, 0)$
Domain: all real numbers
Range: $[-1, 1]$
Asymptotes: none
Period: 2π
Symmetry: odd
one-to-one? No
Partial inverse: $\arcsin x$

Cosine function: $f(x) = \cos x$



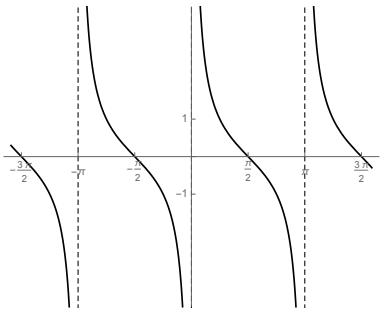
Graph: wave passing through $(0, 1)$
Domain: all real numbers
Range: $[-1, 1]$
Asymptotes: none
Period: 2π
Symmetry: even
one-to-one? No

Tangent function: $f(x) = \tan x$



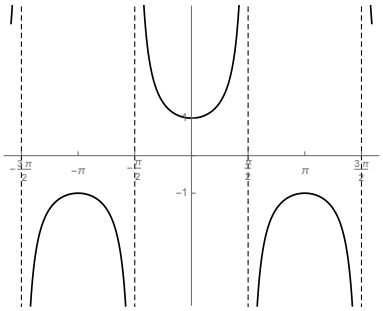
Graph: repeated shapes that look like the graph of x^3, one of which passes through $(0, 0)$
Domain: all real numbers except odd multiples of $\frac{\pi}{2}$
Range: all real numbers
VA: all odd multiples of $\frac{\pi}{2}$
HA: none
Period: π
Symmetry: odd
one-to-one? No
Partial inverse: $\arctan x$

Cotangent function: $f(x) = \cot x$



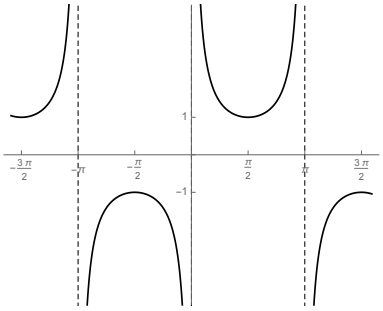
Graph: repeated shapes that look like the graph of x^3 flipped over, one of which passes through $\left(\frac{\pi}{2}, 0\right)$
Domain: all real numbers except multiples of π
Range: all real numbers
VA: all multiples of π
HA: none
Period: π
Symmetry: odd
one-to-one? No

Secant function: $f(x) = \sec x$



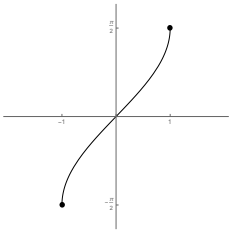
Graph: repeated shapes that look like parabolas, just touching the tops/bottoms of $f(x) = \cos x$
Domain: all real numbers except odd multiples of $\frac{\pi}{2}$
Range: $(-\infty, -1] \cup [1, \infty)$
VA: all odd multiples of $\frac{\pi}{2}$
HA: none
Period: 2π
Symmetry: even
one-to-one? No

Cosecant function: $f(x) = \csc x$



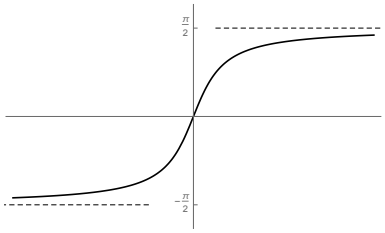
Graph: repeated shapes that look like parabolas, just touching the tops/bottoms of $f(x) = \sin x$
Domain: all real numbers except multiples of π
Range: $(-\infty, -1] \cup [1, \infty)$
VA: all multiples of π
HA: none
Period: 2π
Symmetry: odd
one-to-one? No

Arcsine (a.k.a. inverse sine function): $f(x) = \arcsin x$ (a.k.a. $f(x) = \sin^{-1} x$)



Graph: looks like graph of x^3, goes from $(-1, -\frac{\pi}{2})$ to $(0, 0)$ to $(1, \frac{\pi}{2})$ (graph does not continue to $\pm\infty$)
Domain: $[-1, 1]$
Range: $[-\frac{\pi}{2}, \frac{\pi}{2}]$
Symmetry: odd
Asymptotes: none
Period: none
one-to-one? Yes
Inverse: $\sin x$

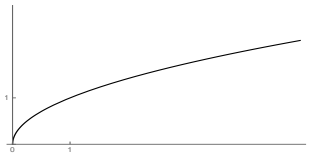
Arctangent (a.k.a. inverse tangent function): $f(x) = \arctan x$ (a.k.a. $f(x) = \tan^{-1} x$)



Graph: looks like one piece of $\tan x$, but flipped over; goes through $(0, 0)$; goes up from left to right
Domain: all real numbers
Range: $(-\frac{\pi}{2}, \frac{\pi}{2})$
Symmetry: odd
HA: $y = \frac{-\pi}{2}$ (to the left); $y = \frac{\pi}{2}$ (to the right)
VA: none
Period: none
one-to-one? Yes
Inverse: $\tan x$

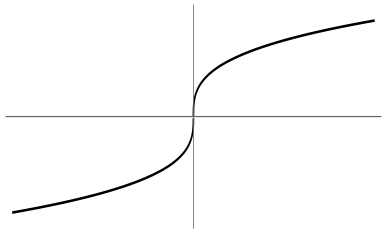
VI. Root functions

Even Root functions: $f(x) = \sqrt[n]{x} = x^{1/n}$ (where $n \geq 2$ is an even number)



Graph: starts at $(0, 0)$, goes up and to the right like the left part of a frown
Domain: $[0, \infty)$
Range: $[0, \infty)$
Asymptotes: none
one-to-one? Yes
Inverse: x^n

Odd Root functions: $f(x) = \sqrt[n]{x} = x^{1/n}$ (where $n \geq 3$ is an odd number)



Graph: looks somewhat like $\arctan x$, but doesn't have a HA (keeps going northeast/southwest rather than east/west)
Domain: all real numbers
Range: all real numbers
Asymptotes: none
one-to-one? Yes
Inverse: x^n