1. Compute the following limits:

   (a) $\lim_{x\to 3} \frac{2x^2-18}{x^2-x-6}$

   (b) $\lim_{x\to -4} \frac{x^2+3x+4}{x-4}$

   (c) $\lim_{x\to \infty} \frac{3x^2+2x-1}{x^4-5x+2}$

   (d) $\lim_{x\to 2} \frac{2x+1}{(x-2)^2}$

2. Compute the indicated derivatives:

   (a) Find $\frac{dy}{dx}$ if $y = 3x^7 - 4x^3 + 5x - 2$.

   (b) Find $f'(2)$ if $f(x) = (4x+1)^{7/2}$.

   (c) Find $f''(x)$ if $f(x) = \frac{3}{x} - \frac{2}{x^3}$.

   (d) Find $f'(x)$ if $f(x) = \frac{5x-3x^2}{2x^4+3}$.

   (e) Find the derivative of $f(x) = (3x^2 - \sqrt{x})(7 - 2x^4 + x)$.

3. (a) Find the slope of the line tangent to the curve $x^2y^3 + 3x = 2y + 5$ at the point $(1, 2)$.

   (b) The number of grams of a compound formed during a chemical reaction t seconds after the reaction starts is $f(t) = \frac{2t}{t+1}$. Find the instantaneous rate of change of the amount of compound with respect to time 4 seconds after the reaction starts.

   (c) Suppose that a bee is flying back and forth such that at time t, its position is given by $f(t) = 2t^3 - 10t^2 + 4t - 2$.

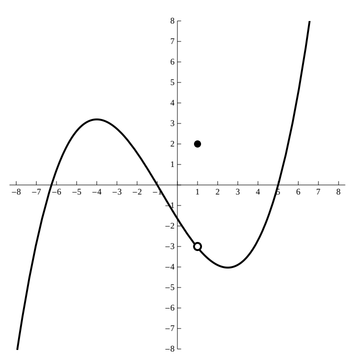
      i. Find the velocity of the bee at time t.

      ii. At time $t = 2$, is the bee moving forwards or backwards? Give a reason for your answer.

      iii. At time $t = 2$, is the bee slowing down or speeding up? Give a reason for your answer.

      iv. Find t such that at time t, the acceleration of the bee is 10.

4. Consider the following graph of a function f, whose formula you do not know:



Based on the graph, answer the following questions.

   (a) Find $\lim_{x\to 1} f(x)$.

   (b) Find $f(1)$.

   (c) On the graph above, sketch the line which is tangent to f at $x = 4$.

   (d) Is $f'(-2)$ positive, negative or zero? Explain your answer.

   (e) Find a value of x for which $f'(x) = 0$ (a reasonable estimate is fine).

   (f) Is $f''(3)$ positive, negative or zero? Explain your answer.

1. (a) $\lim_{x\to 3} \frac{2x^2-18}{x^2-x-6} = \lim_{x\to 3} \frac{2(x+3)(x-3)}{(x-3)(x+2)} = \lim_{x\to 3} \frac{2(x+3)}{(x+2)} = \frac{2(3+3)}{(3+2)} = \frac{12}{5} = 2.4$

   (b) $\lim_{x\to -4} \frac{x^2+3x+4}{x-4} = \frac{16-12+4}{-8} = -1$

   (c) $\lim_{x\to\infty} \frac{3x^2+2x-1}{x^4-5x+2} = \lim_{x\to\infty} \frac{x^4\left(\frac{3}{x^2}+\frac{2}{x^3}-\frac{1}{x^4}\right)}{x^4\left(1-\frac{5}{x^3}+\frac{2}{x^4}\right)} = \lim_{x\to\infty} \frac{\frac{3}{x^2}+\frac{2}{x^3}-\frac{1}{x^4}}{1-\frac{5}{x^3}+\frac{2}{x^4}} = \lim_{x\to\infty} \frac{0+0-0}{1-0+0} = \frac{0}{1} = 0$

   (d) $\lim_{x\to 2} \frac{2x+1}{(x-2)^2} = \frac{7}{0} = \infty$

2. (a) $\frac{dy}{dx} = 21x^6 - 12x^2 + 5$

   (b) By the Chain Rule, $f'(x) = \frac{7}{2}(4x+1)^{5/2} \cdot 4$ so $f'(2) = 14(9^{5/2}) = 3402$.

   (c) First, $f(x) = 3x^{-1} - 2x^{-3}$ so $f'(x) = -3x^{-2} + 6x^{-4}$ so $f''(x) = 6x^{-3} - 24x^{-5}$.

   (d) By the Quotient Rule, $f'(x) = \frac{(5-6x)(2x^4+3)-(8x^3)(5x-3x^2)}{(2x^4+3)^2}$.

   (e) By the Product Rule, $f'(x) = (6x - \frac{1}{2}x^{-1/2})(7-2x^4+x) + (-8x^3+1)(3x^2-\sqrt{x})$.

3. (a) Differentiate implicitly to obtain $2xy^3 + x^2 3y^2 \frac{dy}{dx} + 3 = 2\frac{dy}{dx}$. Now plut in $x = 1, y = 2$ to obtain $2(1)(2^3) + (1^2)3(2^2)\frac{dy}{dx} + 3 = 2\frac{dy}{dx}$, i.e. $19 + 12\frac{dy}{dx} = 2\frac{dy}{dx}$. Solve for $\frac{dy}{dx}$ to obtain $\frac{dy}{dx} = \frac{-19}{10} = -1.9$.

   (b) By the Quotient Rule, $f'(t) = \frac{2(t+1)-1(2t)}{(t+1)^2}$ so the rate of change is $f'(4) = \frac{10-8}{5^2} = \frac{2}{25} = .08$.

   (c) i. $f'(t) = 6t^2 - 20t + 4$

      ii. The velocity at time 2 is $f'(2) = -12$; since this is negative the bee is moving backwards.

      iii. First, $f''(t) = 12t - 20$. The acceleration at time 2 is $f''(2) = 4$, so the bee is speeding up since the acceleration is positive.

      iv. $f''(t) = 12t - 20 = 10$ when $t = \frac{30}{12} = 2.5$.

4. (a) $\lim_{x\to 1} f(x) = -3$, the $y-$coordinate of the hole.

   (b) $f(1) = 2$, the $y-$coordinate of the dot.

   (c) The tangent line should have positive slope and just touch the graph at $x = 4$.

   (d) Negative, because the tangent line has negative slope at $x = -2$ (also because the function is decreasing there).

   (e) $x \approx -4$ or $x \approx 2.5$

   (f) Positive, since the graph of f is concave up at $x = 3$ (the graph looks like part of a $\bigcup$).