Name:

Directions: This exam has seven questions plus a bonus question, spread across eleven pages (not counting this cover page). Answers must be justified appropriately on these pages; show all work and clearly mark your final answers. You may use a calculator, but you may not use a computer, notes or other study aids.

Grading:

Problem	Points Possible	Points Earned
1	24	
2	80	
3	32	
4	24	
5	16	
6	12	
7	12	
Bonus		
Total	200	

1. (8 pts each) Compute any **three** of the following **four** limits. If it isn't clear from your work which four you want me to grade, draw an X through the problem you want me to omit (otherwise, I will grade the first four integrals).

   (a)
   $$\lim_{x\to 4} \frac{(x+2)(x+1)}{x+4}$$

   (b)
   $$\lim_{x\to 2} \frac{x^2+7x-18}{x^2-3x+2}$$

   (c)
   $$\lim_{x\to \infty} \frac{3x^2+4x-1}{6x^2+2}$$

   (d)
   $$\lim_{x\to 3} \frac{x+3}{(x-3)^4}$$

2. (8 pts each) Solve any **ten** of the following **eleven** problems. If it isn't clear from your work which four you want me to grade, draw an X through the problem you want me to omit (otherwise, I will grade the first four integrals).

   (a) Find the derivative of $f(x) = 4x^8 - 3x^4 + 2$.

   (b) Find the slope of the line tangent to $f(x) = (4x^2 + 1)^9$ when $x = 1$.

   (c) Find the instantaneous rate of change of y with respect to x at the point $(1, \frac{1}{11})$ if $y = \frac{2x^2-1}{8x^3+x+2}$.

(d) If the position of an object at time t is given by $f(t) = t - 2\sqrt{t} + \frac{5}{t^2}$, find the velocity of the object at time t.

(e) Compute $\frac{d}{dt}(3\sin x + 2e^x)$.

(f) Find $\frac{dy}{dx}$ if $y = x^4 e^{3x}$.

(g) Find the second derivative of $f(x) = 4\ln(x+2) - \cos 5x$.

(h) Find du if $u = 4\tan 3x$.

(i) Find the derivative of $f(x) = 2\cos^3 x$.

(j) Find $f'(x)$ if $f(x) = x^{-2/3}\sec(x^4 - 1)$.

(k) Find $f'(2)$ if $f(x) = 2x^2e^{2x-1}$.

3. (8 pts each) Compute any **four** of the following **five** integrals. If it isn't clear from your work which four you want me to grade, draw an X through the problem you want me to omit (otherwise, I will grade the first four integrals).

(a)
$$\int (x-1)(x^2+2)\,dx$$

(b)
$$\int (3x+4)^8\,dx$$

(c)
$$\int_{-1}^{2} (3+8x+24x^2)\,dx$$

(d)

$$\int \left(\frac{3}{x^5} - 2\sqrt{x} + 5 \right) dx$$

(e)

$$\int_0^4 2\sqrt{x}\sqrt{x^{3/2}+1}\, dx$$

4. (6 pts each) Suppose that an ant is crawling back and forth along a line so that its position at time 0 is 3 and that its velocity at time t is $v(t) = 6t^2 - 8t + 3$.

   (a) Find the acceleration of the ant at time 3.

   (b) Find the time where the acceleration of the ant is zero.

   (c) At time 3, is the ant moving forward or backward? Explain your answer.

   (d) At time 3, what is the position of the ant?

5. The rate at which energy is released in a chemical reaction at time t is given by $E(t) = \frac{4t}{t^2+100}$.

   (a) (12 pts) Find the time at which this rate is maximized.

   (b) (4 pts) Find the maximum rate of the energy release.

6. Suppose you wanted to approximate $x = \ln 5$ using Newton's method.

   (a) (4 pts) Write down an equation (with no logarithm in it) for which x is a solution that could be used in Newton's method.

   (b) (8 pts) Implement Newton's method with initial guess $x_0 = 1$ using the equation you wrote down in part (a). Estimate x by computing x_2.

7. (12 pts) Suppose a bird is perched along the top of a pole which is 6 ft high. A worm, which is initially at the base of the pole, begins crawling directly away from the pole at a constant rate of .01 feet per second. When the worm has crawled 8 feet, what is the rate of change of the distance from the worm to the bird?

8. (**Bonus**) This problem contains some integrals whose computation rules we did **not** discuss in class. However, based on what we learned about calculus in general, you have enough information to figure out what these integrals are. Evaluate them:

   (a)
   $$\int \frac{1}{x}\,dx$$

   (b)
   $$\int \cos x\,dx$$

   (c)
   $$\int e^x\,dx$$