

Here's an example of a *.tex file. This is not supposed to be sensible mathematics, but merely a bunch of code that produces the document on page 3:

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\documentclass[10pt]{article}
\usepackage{url,amssymb,amsmath,amscd,graphicx,fancyhdr, xypic, xy,amsthm}
\usepackage[top = 1.5 in, bottom = 1 in, left = 1.25 in, right = 1.25 in]{geometry}

\pagestyle{fancy} \lhead{\textsc{Sample \LaTeX\, file}} \chead{\textsc{D. McClendon}}
\rhead{\textsc{Page \thepage}} \lfoot{} \cfoot{}\rfoot{}

\begin{document}

% Anything typed after a % until the next carriage return is treated as
% a 'comment' which doesn't show up in the actual document.

\begin{enumerate}
\item Consider the matrix
\[
A = \left(
\begin{array}{ccc}
1 & -2 & 2 \\
0 & 1 & 1 \\
-1 & 3 & 1
\end{array}
\right).
\]
Find the inverse of  $A$ , and use your answer to find the solution
set of the system  $A\textbf{x} = (0,1,1)$ .

\item For each of the following linear transformations  $T : V_1$ 
 $\rightarrow V_2$ : (i) give the standard matrix of the linear
transformation; (ii) determine whether or not  $T$  is injective;
and (iii) determine whether or not  $T$  is surjective.

\begin{enumerate}
\item  $V_1 = V_2 = \mathbb{P}_3$ ;  $T(f) = f'' - f$ .

\item  $V_1 = \mathbb{R}^2$ ;  $V_2 = \mathbb{R}^4$ ;  $T$  is the
linear transformation satisfying  $T(1,0) = (1,2,-1,0)$  and
 $T(1,1) = (2,6,-3,-2)$ .
\end{enumerate}

\item
\begin{enumerate}
\item Let  $A$  be a  $6 \times 3$  real
matrix of rank 3, and let  $B$  be a  $3 \times 3$  matrix of rank
3. Prove that the matrix  $AB$  has rank 3.

\item Let  $V$  be a vector space with inner product  $\langle, \rangle$ ; let
 $\textbf{w}$  be a nonzero vector in  $V$ . Prove that for any
 $\textbf{x} \in V$ ,
\[
\text{proj}_{\textbf{w}}(\textbf{x}) = \frac{\langle \textbf{x}, \textbf{w} \rangle}{\langle \textbf{w}, \textbf{w} \rangle} \textbf{w}
\]
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\right) = \textrm{proj}_{\textbf{w}} \textbf{x}.

\]

\end{enumerate}

\end{enumerate}

\noindent \textbf{Example:} Find the gradient of $f(x,y,z) = e^x \sin y \cos z$. \\
\textbf{Solution:}

\begin{align*}

\nabla f = \left\langle f_x, f_y, f_z \right\rangle \\\

= \left\langle \sin y \cos z e^x \sin y \cos z, x \cos y \cos z \right. \\
 $e^x \sin y \cos z, -x \sin y \sin z e^x \sin y \cos z$ \left. \right\rangle.

\end{align*}

\vspace{1 in}

This is a direct calculation:

\begin{align*}

r_{XY}(s,t) = Cov(X(s), Y(t)) \\\

= Cov\left(X(s), \frac{1}{t} \int_0^t X(v) \, dv \right) \\\

= \frac{1}{t} \int_0^t Cov(X(s), X(v)) \, dv \\\

= \frac{1}{t} \int_0^t r_X(s,v) \, dv \\\

= \frac{1}{t} \int_0^t \lambda \min(s,v) \, dv \\\

= \left\{ \begin{array}{l} \frac{1}{t} \int_0^s \lambda v \, dv + \frac{1}{t} \int_s^t \lambda s \, dv \text{ if } s \leq t \\ \frac{1}{t} \int_0^t \lambda v \, dv \text{ if } t < s \end{array} \right. \\\

\end{array} \right. \\\

= \left\{ \begin{array}{l} \frac{\lambda s^2}{2t} + \frac{\lambda s(t-s)}{t} \text{ if } s \leq t \\ \frac{\lambda t^2}{2} \text{ if } t < s \end{array} \right. \\\

\end{array} \right. \\\

= \left\{ \begin{array}{l} \frac{\lambda s}{t} (t - \frac{s}{2}) \text{ if } s \leq t \\ \frac{\lambda t}{2} \text{ if } t < s \end{array} \right. \\\

\end{array} \right. \\\

\end{align*}

\end{document}

1. Consider the matrix

$$A = \begin{pmatrix} 1 & -2 & 2 \\ 0 & 1 & 1 \\ -1 & 3 & 1 \end{pmatrix}.$$

Find the inverse of A , and use your answer to find the solution set of the system $A\mathbf{x} = (0, 1, 1)$.

2. For each of the following linear transformations $T : V_1 \rightarrow V_2$: (i) give the standard matrix of the linear transformation; (ii) determine whether or not T is injective; and (iii) determine whether or not T is surjective.

(a) $V_1 = V_2 = \mathbb{P}_3; T(f) = f'' - f$.

(b) $V_1 = \mathbb{R}^2; V_2 = \mathbb{R}^4; T$ is the linear transformation satisfying $T(1, 0) = (1, 2, -1, 0)$ and $T(1, 1) = (2, 6, -3, -2)$.

3. (a) Let A be a 6×3 real matrix of rank 3, and let B be a 3×3 matrix of rank 3. Prove that the matrix AB has rank 3.
- (b) Let V be a vector space with inner product $\langle, \rangle$; let $\mathbf{w}$ be a nonzero vector in V . Prove that for any $\mathbf{x} \in V$,

$$\text{proj}_{\mathbf{w}}(\text{proj}_{\mathbf{w}}\mathbf{x}) = \text{proj}_{\mathbf{w}}\mathbf{x}.$$

Example: Find the gradient of $f(x, y, z) = e^{x \sin y \cos z}$.

Solution:

$$\begin{aligned} \nabla f &= \langle f_x, f_y, f_z \rangle \\ &= \langle \sin y \cos z e^{x \sin y \cos z}, x \cos y \cos z e^{x \sin y \cos z}, -x \sin y \sin z e^{x \sin y \cos z} \rangle. \end{aligned}$$

This is a direct calculation:

$$\begin{aligned} r_{XY}(s, t) &= \text{Cov}(X(s), Y(t)) \\ &= \text{Cov}\left(X(s), \frac{1}{t} \int_0^t X(v) dv\right) \\ &= \frac{1}{t} \int_0^t \text{Cov}(X(s), X(v)) dv \\ &= \frac{1}{t} \int_0^t r_X(s, v) dv \\ &= \frac{1}{t} \int_0^t \lambda \min(s, v) dv \\ &= \begin{cases} \frac{1}{t} \int_0^s \lambda v dv + \frac{1}{t} \int_s^t \lambda s dv & \text{if } s \leq t \\ \frac{1}{t} \int_0^t \lambda v dv & \text{if } t < s \end{cases} \\ &= \begin{cases} \frac{\lambda s^2}{2t} + \frac{\lambda s(t-s)}{t} & \text{if } s \leq t \\ \frac{\lambda t^2}{2t} & \text{if } t < s \end{cases} \\ &= \begin{cases} \frac{\lambda s}{t} \left(t - \frac{s}{2}\right) & \text{if } s \leq t \\ \frac{\lambda t}{2} & \text{if } t < s \end{cases} \end{aligned}$$