The following sum and integral formulas are useful in probability theory.

From time to time, when solving a problem we will obtain the left-hand side of one of these formulas; to proceed with the solution we replace it with the corresponding right-hand side.

Triangular Number Formula: For all $n \in \{1, 2, 3, ...\}$,

$$1 + 2 + 3 + ... + n = \sum_{j=0}^{n} j = \frac{n(n+1)}{2}.$$

Finite Geometric Series Formula: for all $r \in \mathbb{R}$,

$$\sum_{n=0}^{N} r^n = \frac{1 - r^{N+1}}{1 - r}.$$

Infinite Geometric Series Formula: for all $r \in \mathbb{R}$ such that $|r| < 1$,

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r} \qquad \text{and} \qquad \sum_{n=N}^{\infty} r^n = \frac{r^N}{1-r}$$

Derivative of the Geometric Series Formula: for all $r \in \mathbb{R}$ such that $|r| < 1$,

$$\sum_{n=0}^{\infty} n r^n = \frac{r}{(1-r)^2}.$$

Exponential Series Formula: for all $r \in \mathbb{R}$,

$$\sum_{n=0}^{\infty} \frac{r^n}{n!} = e^r.$$

Binomial Theorem: for all $n \in \mathbb{N}$, and all $x, y \in \mathbb{R}$,

$$\sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k} = (x+y)^n.$$

Vandermonde Identity: for all $n, k, r \in \mathbb{N}$,

$$\sum_{x=0}^{n} \binom{r}{x} \binom{n-r}{k-x} = \binom{n}{k}.$$

Gamma Integral Formula: for all $r > 0$, $\lambda > 0$,

$$\int_0^\infty x^{r-1} e^{-\lambda x}\, dx = \frac{\Gamma(r)}{\lambda^r}.$$

Normal Integral Formula: for all $\mu \in \mathbb{R}$ and all $\sigma > 0$,

$$\int_{-\infty}^{\infty} \exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right) dx = \sigma\sqrt{2\pi}.$$

Beta Integral Formula: for all $r > 0$, $\lambda > 0$,

$$\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\, dx = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}.$$