First of all, this document has text in three different fonts. Anything written in this font is simply commentary what is going on. `Anything written in this font refers to what you would type in your LaTeX editor.` Anything written in this font is output you would get in your pdf file.

1 Typesetting in text mode

1.1 Quote marks

To surround something with quotes, use " (the button above the tab key) before and '' (two apostrophes) after the item.

1.2 New paragraphs

To start a new paragraph, use two returns in your editor at the end of a sentence. For example, if my editor looks like:

```
This is a random long sentence with a bunch of random long phrases and
this sentence ends here.

The next sentence is here.
```

then the document that is produced will look like:

> This is a random long sentence with a bunch of random long phrases and this sentence ends here. The next sentence is here.

You can use a double backslash to force a carriage return. For example, if the editor looks like:

```
This is a short sentence. \\
The next sentence is here.
```

then the `\\` produces a carriage return, but not a new paragraph (and the next line is not indented), i.e. the document that is produced will look like:

> This is a short sentence.
The next sentence is here.

1.3 Indents

Every time you start a new paragraph, LaTeX will automatically indent the first sentence. If you don't want the indent, insert the command `\noindent` before the start of the paragraph, i.e. if the editor looks like

```
This is a short sentence. \\
```

then you will see the following sentence, which is indented:

> This is a short sentence.

but if the editor looks like

```
\noindent This is a short sentence. \\
```

you will see the following sentence, which is not indented:

This is a short sentence.

1.4   Skipping lines

If the editor looks like:

```
This is a random long sentence with a bunch of random long phrases and
this sentence ends here. \\

The next sentence is here.
```

then the document that is produced will look like:

This is a random long sentence with a bunch of random long phrases and this sentence ends here.

The next sentence is here.

1.5   Leaving space

Suppose you want to leave some space (to draw a figure in a document, etc.). To do this, type something like this:

```
This is a random long sentence with a bunch of random long phrases and
this sentence ends here. \vspace{2 in}

The next sentence is here.
```

This will leave a 2 inch (measured vertically) space for you. You can use any number in place of the 2, and you can use centimeters (type cm) or other units of measurement as well.

1.6   New pages

Typing \newpage will make LaTeX put whatever follows the \newpage on the next page.

1.7   Footnotes

Typing \footnote{ } will make a footnote where the footnote is whatever is inside the braces.

1.8   Centering

Type this:

```
\begin{center}
Centered Text
```

```
\end{center}
```

to get this:

Centered Text

1.9 Font formatting

FONT STYLE	TYPE THIS	TO PRODUCE THIS
Regular text	`word`	word
Italics	`\emph{word}`	*word*
Bold	`\bf{word word}`	**word word**
Underline	`\underline{word}`	<u>word</u>
Small caps	`\textsc{Word}`	WORD
Serif font	`\textsf{Word}`	Word

Different sizes	`Word`
	`\small{Word}`
	`\large{Word}`
	`\Large{Word}`
	`\LARGE{Word}`
		Word
		Word
		Word
		Word
		Word

Combining these	`\textsc{Word \textbf{Word` `\underline{Word}}}`	WORD **Word <u>Word</u>**

1.10 Lists

LaTeX creates lists easily. If you type:

```
\begin{enumerate}
\item Here is the first item.
\item Here is another item.
\item Here is yet another item.  Each of these items could hypothetically go on
for pages and pages.
\end{enumerate}
```

then you will see:

1. Here is the first item.

2. Here is another item.

3. Here is yet another item. Each of these items could hypothetically go on for pages and pages.

If you want to enumerate by something other than roman numbers, you can type (for example)

```
\begin{enumerate}
\item[i.] Here is the first item.
\item[ii.] Here is another item.
\item[iii.] Here is yet another item.  Each of these items could hypothetically go
on for pages and pages.
\end{enumerate}
```

and obtain:

i. Here is the first item.

ii. Here is another item.

iii. Here is yet another item. Each of these items could hypothetically go on for pages and pages.

To make a bulleted list of items, type

```
\begin{itemize}
\item Here is the first item.
\item Here is another item.
\item Here is yet another item.  Each of these items could hypothetically go
on for pages and pages.
\end{itemize}
```

to obtain

- Here is the first item.

- Here is another item.

- Here is yet another item. Each of these items could hypothetically go on for pages and pages.

Another thing you can do is this:

```
\begin{description}
\item[Label 1 blah blah] Here is the first item.
\item[Label 2] Here is another item.
\end{description}
```

which produces

Label 1 blah blah Here is the first item.

Label 2 Here is another item.

You can put these commands inside of one another to make outlines, etc. For example:

```
\begin{enumerate}
\item Warm colors
\begin{enumerate}
\item Red
\item Orange
\end{enumerate}
\item Cool colors
\begin{itemize}
\item Purple
\item Blue
\end{itemize}
\end{enumerate}
```

produces:

1. Warm colors

    (a) Red
    (b) Orange

2. Cool colors

    - Purple
    - Blue

2   Mathematical typesetting

2.1   General principles: math mode vs. text mode

When you type in a LaTeX file, you toggle between text mode and math mode. To toggle between the two modes within a paragraph, use a $. To produce a centered line of mathematical symbols, surround the symbols with \[and \] (without using a $). For example, if you type:

```
The integral $\int x^2 \, dx = \frac{x^3}3 + C$ is an example of the Fundamental
Theorem of Calculus.
```

then the corresponding text that is produced is

The integral $\int x^2 \, dx = \frac{x^3}{3} + C$ is an example of the Fundamental Theorem of Calculus.

If you type:

```
Here's a sentence and inside this sentence I will type \[ x = 7 + a_n\] and then the
sentence goes on.
```

then the corresponding text that is produced is

Here's a sentence and inside this sentence I will type

$$x = 7 + a_n$$

and then the sentence goes on.

Warning: After typing a line or two of math symbols, compile your document to see if it worked. Then type some more and compile again, etc.

2.2   Letters in mathematics

This chart tells you how to get various fonts in math mode; note that while in math mode there are no spaces unless you use some kind of special command (see the first line of the chart below):

Type of letters	Type this (while in math mode)	to produce this
	`text is here`	$textishere$
Regular text	`\textrm{text is here}`	text is here
Roman letters	`a,b,c,d,...,x,y,z`	$a, b, c, ..., x, y, z$
Roman letters	`A,B,C,...,X,Y,Z`	$A, B, C, ..., X, Y, Z$
boldface letters (vectors)	`\mathbf{v}, \mathbf{X}`	$\mathbf{v}, \mathbf{X}$
Sets of numbers	`\mathbb{R}, \mathbb{C}, \mathbb{Z}`	$\mathbb{R}, \mathbb{C}, \mathbb{Z}$
Script capitals	`\mathcal{A}, \mathcal{Q}, \mathcal{Z}`	$\mathcal{A}, \mathcal{Q}, \mathcal{Z}$
Greek letters	`\alpha, \delta, \epsilon, \zeta, \pi`	$\alpha, \delta, \epsilon, \zeta, \pi$
Capital Greek letters	`\Pi, \Sigma, \Phi, \Upsilon`	$\Pi, \Sigma, \Phi, \Upsilon$

2.3   Modified letters

Type of letters	Type this (while in math mode)	to produce this
Regular text	`x`	x
Primes	`f'(x)`	$f'(x)$

Bars	`\bar{x}`
	`\overline{x + 3y}`
	   Note: use `\overline` to make a bar
	   over a long expression
		$\bar{x}$
		$\overline{x + 3y}$

Tildes	`\widetilde{x}`	$\widetilde{x}$

Hats	`\hat{x}`
	`\widehat{x - y}`
	   Note: use `\widehat` to make a hat
	   over a long expression
		$\hat{x}$
		$\widehat{x - y}$

Exponents	`x^4`
	`x^13`
	`x^{13}`
	`x^{2^n}`
	`x^*`
	   Note the difference between the
	   second and third commands
		x^4
		x^13
		x^{13}
		x^{2^n}
		x^*

Subscripts	`x_4`
	`x_13`
	`x_{13}`
	`x_{2_n}`
	`x_2^4`
	`x_{2^4}`
		x_4
		x_13
		x_{13}
		x_{2_n}
		x_2^4
		x_{2^4}

Fractions	`\frac 12`
	`\frac 1 32`
	`\frac 1{32}`
	`\frac {x - 3x^2y}{32 x^{4x} - 3}`
		$\frac{1}{2}$
		$\frac{1}{3}2$
		$\frac{1}{32}$
		$\frac{x-3x^2y}{32x^{4x}-3}$

2.4 Symbols

Symbol	Type this (while in math mode)	to produce this
Arithmetic	`+,-,\times,\div,\cdot,/` `\sqrt{x}, \sqrt[4]{x + 3}`	$+,-,\times,\div,\cdot,/$ $\sqrt{x}, \sqrt[4]{x+3}$
Equalities	`=, \neq, \sim, \cong, \approx, \equiv`	$=,\neq,\sim,\cong,\approx,\equiv$
Inequalities	`<,>,\geq,\leq`	$<,>,\geq,\leq$

Set relations	`\subset, \subseteq, \supset, \supseteq`
	`\in, \notin, \ni`
	`\cup, \cap, \bigcup, \bigcap, \vee, \bigvee`
		$\subset,\subseteq,\supset,\supseteq$
		$\in,\notin,\ni$
		$\cup,\cap,\bigcup,\bigcap,\vee,\bigvee$

Infinity	`\infty`	∞

Braces
Parenthesis and brackets
Large parenthesis or brackets
Absolute value signs
	`\{, \}`
	`(, [, ), ]`
	`\left( \frac x2 \right)`
	`|`
		$\{,\}$
		$(,[,),]$
		$\left(\frac{x}{2}\right)$
		$|$

Partial derivatives	`\partial, \nabla`	∂, ∇
Dots	`\cdots, \ddots, \vdots`	$\cdots, \ddots, \vdots$
Integrals	`\int` `\int_a^{2b} x^2 dx`	$\int$ $\int_a^{2b} x^2 dx$

Arrows	`\rightarrow, \leftarrow, \mapsto, \to`
	`\Leftarrow, \Leftrightarrow`
	`f : A \to \mathbb{R}`
	`\lim_{x \to 3} f(x)`
		$\rightarrow,\leftarrow,\mapsto,\to$
		$\Leftarrow,\Leftrightarrow$
		$f:A\to\mathbb{R}$
		$\lim_{x\to3} f(x)$

Quantifiers	`\forall, \exists`	$\forall, \exists$
Factorials	`!`	$!$
Sums	`\sum_{n = 1}^\infty`	$\sum_{n=1}^\infty$
Composition	`f \circ g`	$f \circ g$

2.5 Common math words

In math mode, to get normal (non-italicized) letters you usually have to type `\textrm{ }`. But many common text expressions you would use in math expressions can be obtained simply by typing a backslash, followed by the appropriate text. These include:

Type this (while in math mode)	to produce this
`\sin`	sin
`\cos`	cos
`\tan`	tan
`\arctan`	arctan
`\ln`	ln
`\log`	log
`\exp`	exp
`\lim`	lim

2.6 Matrices

To produce a matrix, use the following syntax while in math mode. The number of c's in the first line should be the number of columns in the matrix. You can put in as many rows as you like; separate each entry by an ampersand and separate each row by two backslashes.

```
\left( \begin{array}{ccccc}
a & b & c & d & \int e \, dx \\
3f^2-4f & g_3^2 & h & i & j \\
\end{array} \right)
```

This produces the matrix

$$\begin{pmatrix} a & b & c & d & \int e\,dx \\ 3f^2 - 4f & g_3^2 & h & i & j \end{pmatrix}$$

2.7 Spacing in math mode

To produce small horizontal spaces between characters inside math equations, use combinations of `\,` or `\;` or `\quad` depending on how much space you want.

2.8 Aligned equations

To produce a sequence of equations that are lined up, use the following syntax. **This syntax is not to be used in math mode; do not surround it with \$s.** In particular, LaTeX will arrange the equations so that the symbols following the ampersands are under one another. Use two backslashes at the end of each line:

```
\begin{align*}
(a + b)^2 - ||a||^2 & = a^2 + 2ab + b^2 - ||a||^2 \\
& = a^2 - 2ab + b^2 + 4ab -||a||^2 \\
& = (a - b)^2 + 4ab - ||a||^2 \\
\Rightarrow z & < 35000 \cdot \infty \\
```

`\end{align*}`

produces

$$(a + b)^2 - ||a||^2 = a^2 + 2ab + b^2 - ||a||^2$$
$$= a^2 - 2ab + b^2 + 4ab - ||a||^2$$
$$= (a - b)^2 + 4ab - ||a||^2$$
$$\Rightarrow z < 35000 \cdot \infty$$

2.9   Referring to equations

One really nice thing about LaTeX is that you can "label" equations and refer to them later. For example, suppose you type

```
\begin{equation} \label{myfirstequation}
(a + b)^2 - ||a||^2  = a^2 + 2ab + b^2 - ||a||^2
\end{equation}
```

to produce the equation

$$(a + b)^2 - ||a||^2 = a^2 + 2ab + b^2 - ||a||^2 \tag{1}$$

Notice that this automatically numbers the equation (this one is numbered (1)). Now, to refer to this equation later, you can type something like:

```
Now it follows from equation (\ref{myfirstequation}) that so and so...
```

and this will produce (**after you compile the document twice**)

Now it follows from equation (1) that so and so...

If you go back and change the document, adding more equations earlier on, the numbering will change but all your references will be fine (i.e. if you were to insert another equation earlier, this one would now be numbered 2, and your subsequent sentence would refer to equation 2 as it is supposed to).

3   Example

Here's an example of a LaTeX file. This is not supposed to be sensible mathematics, but merely a bunch of LaTeX which produces the document which follows it:

```
\documentclass[10pt]{article}
\usepackage{url,amssymb,amsmath,amscd,graphicx,fancyhdr, xypic, xy,amsthm}
\usepackage[top = 1.5 in, bottom = 1 in, left = 1.25 in, right = 1.25 in]{geometry}

\pagestyle{fancy} \lhead{\textsc{Sample}} \chead{\textsc{Blah}}
\rhead{\textsc{Page \thepage}} \lfoot{} \cfoot{}\rfoot{}

\begin{document}

%  Anything typed after a % until the next carriage return is treated as
 a 'comment' which doesn't show up in the actual document.

\begin{enumerate}
\item Consider the matrix
\[
A = \left(
\begin{array}{ccc}
  1 & -2 & 2 \\
  0 & 1 & 1 \\
  -1 & 3 & 1 \\
\end{array}
\right).
\]
Find the inverse of $A$, and use your answer to find the solution
set of the system $A\textbf{x} = (0,1,1)$.

\item For each of the following linear transformations $T : V_1
\rightarrow V_2$: (i) give the standard matrix of the linear
transformation; (ii) determine whether or not $T$ is injective;
and (iii) determine whether or not $T$ is surjective.

\begin{enumerate}

\item $V_1 = V_2 = \mathbb{P}_3$; $T(f) = f'' - f$.

\item $V_1 = \mathbb{R}^2$; $V_2 = \mathbb{R}^4$; $T$ is the
linear transformation satisfying $T(1,0) = (1,2,-1,0)$ and
$T(1,1)=(2,6,-3,-2)$.
\end{enumerate}

\item
\begin{enumerate}
\item Let $A$ be a $6 \times 3$ real
matrix of rank $3$, and let $B$ be a $3 \times 3$ matrix of rank
$3$. Prove that the matrix $AB$ has rank $3$.
```

```
\item Let $V$ be a vector space with inner product $<,>$; let
$\textbf{w}$ be a nonzero vector in $V$.  Prove that for any
$\textbf{x} \in V$,
\[
\textrm{proj}_\textbf{w}\left( \textrm{proj}_\textbf{w} \textbf{x}
\right) = \textrm{proj}_\textbf{w} \textbf{x}.
\]
\end{enumerate}

\end{enumerate}

\noindent \textbf{Example:} Find the gradient of $f(x,y,z) = e^{x\sin y \cos z}$. \\
\textbf{Solution:}
\begin{align*}
\nabla f & =  \left<f_x, f_y, f_z \right> \\
& =  \left<\sin y \cos z e^{x \sin y \cos z}, x \cos y \cos z
e^{x \sin y \cos z}, -x \sin y \sin z e^{x \sin y \cos z} \right>.
\end{align*}

\vspace{1 in}

This is a direct calculation:
\begin{align*}
\r_{XY}(s,t) & = & Cov(X(s), Y(t)) \\
& =  Cov\left(X(s), \frac 1t \int_0^t X(v) \, dv \right) \\
& =  \frac 1t \int_0^t Cov(X(s), X(v))\, dv \\
& =  \frac 1t \int_0^t r_X(s,v) \, dv \\
& =  \frac 1t \int_0^t \lambda \min(s,v) \, dv \\
& =  \left\{\begin{array}{ll}
  \frac 1t \int_0^s \lambda v \, dv + \frac 1t \int_s^t \lambda s \, dv & \textrm{ if }s \leq t \\
  \frac 1t \int_0^t \lambda v \, dv & \textrm{ if }t < s \\
\end{array} \right. \\
& =  \left\{\begin{array}{ll}
  \frac{\lambda s^2}{2t} + \frac{\lambda s (t-s)}{t} \, dv & \textrm{ if }s \leq t \\
  \frac {\lambda t^2}{2t} & \textrm{ if }t < s \\
\end{array} \right. \\
& =  \left\{\begin{array}{ll}
  \frac{\lambda s}{t}(t - \frac s2) & \textrm{ if }s \leq t \\
  \frac{\lambda t}{2} & \textrm{ if }t < s \\
\end{array} \right. \\
\end{align*}

\end{document}
```

1. Consider the matrix
$$A = \begin{pmatrix} 1 & -2 & 2 \\ 0 & 1 & 1 \\ -1 & 3 & 1 \end{pmatrix}.$$
   Find the inverse of A, and use your answer to find the solution set of the system $A\mathbf{x} = (0, 1, 1)$.

2. For each of the following linear transformations $T : V_1 \to V_2$: (i) give the standard matrix of the linear transformation; (ii) determine whether or not T is injective; and (iii) determine whether or not T is surjective.

   (a) $V_1 = V_2 = \mathbb{P}_3$; $T(f) = f'' - f$.
   (b) $V_1 = \mathbb{R}^2$; $V_2 = \mathbb{R}^4$; T is the linear transformation satisfying $T(1, 0) = (1, 2, -1, 0)$ and $T(1, 1) = (2, 6, -3, -2)$.

3. (a) Let A be a 6×3 real matrix of rank 3, and let B be a 3×3 matrix of rank 3. Prove that the matrix AB has rank 3.

   (b) Let V be a vector space with inner product $<,>$; let $\mathbf{w}$ be a nonzero vector in V. Prove that for any $\mathbf{x} \in V$,
$$\text{proj}_\mathbf{w} (\text{proj}_\mathbf{w}\mathbf{x}) = \text{proj}_\mathbf{w}\mathbf{x}.$$

Example: Find the gradient of $f(x, y, z) = e^{x \sin y \cos z}$.
Solution:

$$\nabla f = \langle f_x, f_y, f_z \rangle$$
$$= \left\langle \sin y \cos z e^{x \sin y \cos z}, x \cos y \cos z e^{x \sin y \cos z}, -x \sin y \sin z e^{x \sin y \cos z} \right\rangle.$$

This is a direct calculation:

$$
\begin{aligned}
r_{XY}(s, t) &= Cov(X(s), Y(t)) \\
&= Cov\left(X(s), \frac{1}{t} \int_0^t X(v)\, dv\right) \\
&= \frac{1}{t} \int_0^t Cov(X(s), X(v))\, dv \\
&= \frac{1}{t} \int_0^t r_X(s, v)\, dv \\
&= \frac{1}{t} \int_0^t \lambda \min(s, v)\, dv \\
&= \begin{cases} \frac{1}{t} \int_0^s \lambda v\, dv + \frac{1}{t} \int_s^t \lambda s\, dv & \text{if } s \leq t \\ \frac{1}{t} \int_0^t \lambda v\, dv & \text{if } t < s \end{cases} \\
&= \begin{cases} \frac{\lambda s^2}{2t} + \frac{\lambda s(t-s)}{t}\, dv & \text{if } s \leq t \\ \frac{\lambda t^2}{2t} & \text{if } t < s \end{cases} \\
&= \begin{cases} \frac{\lambda s}{t}(t - \frac{s}{2}) & \text{if } s \leq t \\ \frac{\lambda t}{2} & \text{if } t < s \end{cases}
\end{aligned}
$$